

Key Ideas

- A sequence is an ordered list of elements.
- Elements within the range of the sequence are called terms of the sequence.
- To describe any term of a sequence, an expression is used for t_n , where $n \in \mathbb{N}$. This term is called the general term.
- In an arithmetic sequence, each successive term is formed by adding a constant. This constant is called the common difference.

- The general term of an arithmetic sequence is

$$t_n = t_1 + (n - 1)d$$

where t_1 is the first term

n is the number of terms ($n \in \mathbb{N}$)

d is the common difference

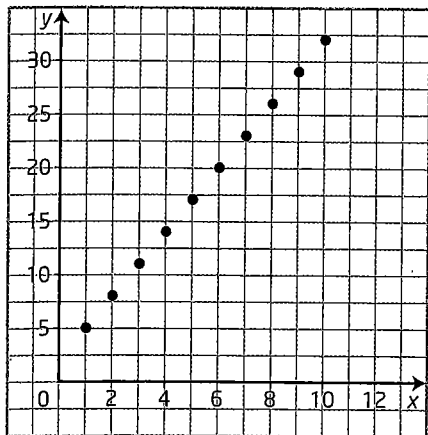
t_n is the general term or n th term

Check Your Understanding

Practise

1. Identify the arithmetic sequences from the following sequences. For each arithmetic sequence, state the value of t_1 , the value of d , and the next three terms.
 - a) 16, 32, 48, 64, 80, ...
 - b) 2, 4, 8, 16, 32, ...
 - c) -4, -7, -10, -13, -16, ...
 - d) 3, 0, -3, -6, -9, ...
2. Write the first four terms of each arithmetic sequence for the given values of t_1 and d .
 - a) $t_1 = 5$, $d = 3$
 - b) $t_1 = -1$, $d = -4$
 - c) $t_1 = 4$, $d = \frac{1}{5}$
 - d) $t_1 = 1.25$, $d = -0.25$
3. For the sequence defined by $t_n = 3n + 8$, find each indicated term.
 - a) t_1
 - b) t_7
 - c) t_{14}
4. For each arithmetic sequence determine the values of t_1 and d . State the missing terms of the sequence.
 - a) ■, ■, ■, 19, 23
 - b) ■, ■, 3, $\frac{3}{2}$
 - c) ■, 4, ■, ■, 10
5. Determine the position of the given term to complete the following statements.
 - a) 170 is the ■th term of -4, 2, 8, ...
 - b) -14 is the ■th term of $2\frac{1}{5}$, 2, $1\frac{4}{5}$, ...
 - c) 97 is the ■th term of -3, 1, 5, ...
 - d) -10 is the ■th term of 14, 12.5, 11, ...
6. Determine the second and third terms of an arithmetic sequence if
 - a) the first term is 6 and the fourth term is 33
 - b) the first term is 8 and the fourth term is 41
 - c) the first term is 42 and the fourth term is 27

7. The graph of an arithmetic sequence is shown.



- What are the first five terms of the sequence?
- Write the general term of this sequence.
- What is t_{50} ? t_{200} ?
- Describe the relationship between the slope of the graph and your formula from part b).
- Describe the relationship between the y -intercept and your formula from part b).

Apply

- Which arithmetic sequence(s) contain the term 34? Justify your conclusions.
 - $t_n = 6 + (n - 1)4$
 - $t_n = 3n - 1$
 - $t_1 = 12, d = 5.5$
 - 3, 7, 11, ...
- Determine the first term of the arithmetic sequence in which the 16th term is 110 and the common difference is 7.
- The first term of an arithmetic sequence is $5y$ and the common difference is $-3y$. Write the equations for t_n and t_{15} .
- The terms $5x + 2$, $7x - 4$, and $10x + 6$ are consecutive terms of an arithmetic sequence. Determine the value of x and state the three terms.

- The numbers represented by x , y , and z are the first three terms of an arithmetic sequence. Express z in terms of x and y .
- Each square in this pattern has a side length of 1 unit. Assume the pattern continues.

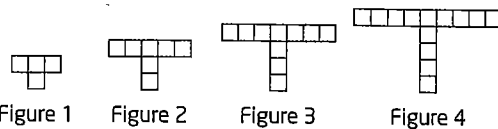


Figure 1 Figure 2 Figure 3 Figure 4

- Write an equation in which the perimeter is a function of the figure number.
 - Determine the perimeter of Figure 9.
 - Which figure has a perimeter of 76 units?
- The Wolf Creek Golf Course, located near Ponoka, Alberta, has been the site of the Canadian Tour Alberta Open Golf Championship. This tournament has a maximum entry of 132 players. The tee-off times begin at 8:00 and are 8 min apart.
 - The tee-off times generate an arithmetic sequence. Write the first four terms of the arithmetic sequence, if the first tee-off time of 8:00 is considered to be at time 0.
 - Following this schedule, how many players will be on the course after 1 h, if the tee-off times are for groups of four?
 - Write the general term for the sequence of tee-off times.
 - At what time will the last group tee-off?
 - What factors might affect the prearranged tee-off time?

Did You Know?

The first championship at Wolf Creek was held in 1987 and has attracted PGA professionals, including Mike Weir and Dave Barr.

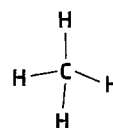
15. Lucy Ango'yuaq, from Baker Lake, Nunavut, is a prominent wall hanging artist. This wall hanging is called *Geese and Ulus*. It is 22 inches wide and 27 inches long and was completed in 27 days. Suppose on the first day she completed 48 square inches of the wall hanging, and in the subsequent days the sequence of cumulative areas completed by the end of each day produces an arithmetic sequence. How much of the wall hanging did Lucy complete on each subsequent day? Express your answer in square inches.



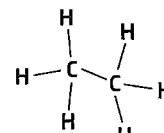
The Inuktitut syllabics appearing at the bottom of this wall hanging spell the artist's name. For example, the first two syllabics spell out Lu-Si.

16. Susan joined a fitness class at her local gym. Into her workout, she incorporated a sit-up routine that followed an arithmetic sequence. On the 6th day of the program, Susan performed 11 sit-ups. On the 15th day she did 29 sit-ups.
- Write the general term that relates the number of sit-ups to the number of days.
 - If Susan's goal is to be able to do 100 sit-ups, on which day of her program will she accomplish this?
 - What assumptions did you make to answer part b)?

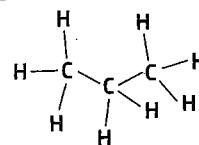
17. Hydrocarbons are the starting points in the formation of thousands of products, including fuels, plastics, and synthetic fibres. Some hydrocarbon compounds contain only carbon and hydrogen atoms. Alkanes are saturated hydrocarbons that have single carbon-to-carbon bonds. The diagrams below show the first three alkanes.



Methane



Ethane



Propane

- a) The number of hydrogen atoms compared to number of carbon atoms produces an arithmetic sequence. Copy and complete the following chart to show this sequence.

Carbon Atoms	1	2	3	4
Hydrogen Atoms	4			

- b) Write the general term that relates the number of hydrogen atoms to the number of carbon atoms.
- c) Hectane contains 202 hydrogen atoms. How many carbon atoms are required to support 202 hydrogen atoms?

18. The multiples of 5 between 0 and 50 produces the arithmetic sequence 5, 10, 15, ..., 45. Copy and complete the following table for the multiples of various numbers.

Multiples of	28	7	15
Between	1 and 1000	500 and 600	50 and 500
First Term, t_1			
Common Difference, d			
nth Term, t_n			
General Term			
Number of Terms			

Answers

Chapter 1 Sequences and Series

1.1 Arithmetic Sequences, pages 16 to 21

1. a) arithmetic sequence: $t_1 = 16$, $d = 16$; next three terms: 96, 112, 128
 b) not arithmetic
 c) arithmetic sequence: $t_1 = -4$, $d = -3$; next three terms: -19, -22, -25
 d) arithmetic sequence: $t_1 = 3$, $d = -3$; next three terms: -12, -15, -18
2. a) 5, 8, 11, 14 b) -1, -5, -9, -13
 c) $4, \frac{21}{5}, \frac{22}{5}, \frac{23}{5}$ d) 1.25, 1.00, 0.75, 0.50
3. a) $t_1 = 11$ b) $t_7 = 29$ c) $t_{14} = 50$
4. a) 7, 11, 15, 19, 23; $t_1 = 7$, $d = 4$
 b) $6, \frac{9}{2}, 3, \frac{3}{2}$; $t_1 = 6$, $d = -\frac{3}{2}$
 c) 2, 4, 6, 8, 10; $t_1 = 2$, $d = 2$
5. a) 30 b) 82 c) 26 d) 17
6. a) $t_2 = 15$, $t_3 = 24$ b) $t_2 = 19$, $t_3 = 30$
 c) $t_2 = 37$, $t_3 = 32$
7. a) 5, 8, 11, 14, 17 b) $t_n = 3n + 2$
 c) $t_{50} = 152$, $t_{200} = 602$
 d) The general term is a linear equation of the form $y = mx + b$, where $t_n = y$ and $n = x$. Therefore, $t_n = 3n + 2$ has a slope of 3.
 e) The constant value of 2 in the general term is the y -intercept of 2.
8. A and C; both sequences have a natural-number value for n .
9. 5
10. $t_n = -3n + 8$; $t_{15} = -37$
11. $x = -16$; first three terms: -78, -116, -154
12. $z = 2y - x$
13. a) $t_n = 6n + 4$ b) 58
 c) 12
14. a) 0, 8, 16, 24
 b) 32 players
 c) $t_n = 8n - 8$
 d) 12:16
 e) Example: weather, all foursomes starting on time, etc.
15. 21 square inches
16. a) $t_n = 2n - 1$ b) 51st day
 c) Susan continues the program until she accomplishes her goal.
17. a)

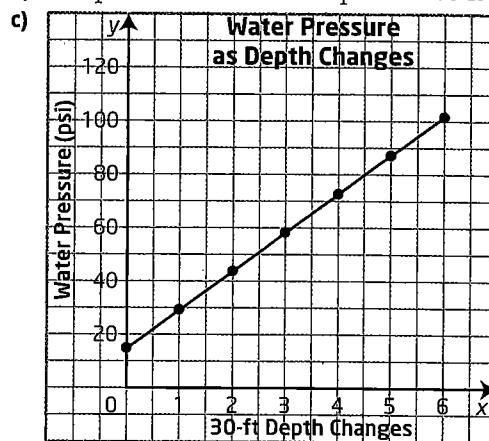
Carbon Atoms	1	2	3	4
Hydrogen Atoms	4	6	8	10

 b) $t_n = 2n + 2$ or $H = 2C + 2$
 c) 100 carbon atoms

18.

Multiples of	28	7	15
Between	1 and 1000	500 and 600	50 and 500
First Term, t_1	28	504	60
Common Difference, d	28	7	15
n th Term, t_n	980	595	495
General Term	$t_n = 28n$	$t_n = 7n + 497$	$t_n = 15n + 45$
Number of Terms	35	14	30

19. a) 14.7, 29.4, 44.1, 58.8; $t_n = 14.7n$, where n represents every increment of 30 ft in depth.
 b) 490 psi at 1000 ft and 980 psi at 2000 ft



- d) 14.7 psi
- e) 14.7
- f) The y -intercept represents the first term of the sequence and the slope represents the common difference.
20. Other lengths are 6 cm, 12 cm, and 18 cm. Add the four terms to find the perimeter. Replace t_2 with $t_1 + d$, t_3 with $t_1 + 2d$, and t_4 with $t_1 + 3d$. Solve for d .
21. a) 4, 8, 12, 16, 20 b) $t_n = 4n$
 c) 320 min
22. -29 beekeepers
23. 5.8 million carats. This value represents the increase of diamond carats mined each year.
24. 1696.5 m
25. a) 13:54, 13:59, 14:04, 14:09, 14:14; $t_1 = 13:54$, $d = 0:05$
 b) $t_n = 0:05n + 13:49$
 c) Assume that the arithmetic sequence of times continues.
 d) 15:49