

1.2 Arithmetic Series

An **Arithmetic Series** is the Sum of the terms in an Arithmetic Sequence.

$$S_n = \frac{n}{2}(t_1 + t_n)$$

t_1 = first term
 n = the number of terms
 t_n = the n^{th} term (sum up to)
 S_n = sum of the first n terms

$$S_n = \frac{n}{2}[2t_1 + (n-1)d]$$

t_1 = first term
 d = common difference
 n = the number of terms
 S_n = sum of the first n terms

Example 1: What is the sum of the first 7 terms of the following arithmetic series: $5 + 8 + 11 + 14 + \dots$?

$$t_1 = 5$$

$$d = 3$$

$$n = 7$$

$$S_n = \frac{n}{2}(t_1 + t_n) \quad \text{or} \quad S_n = \frac{n}{2}[2t_1 + (n-1)d]$$

we don't know
this term.
We must use the
other formula.

$$S_7 = \frac{7}{2}[2(5) + (7-1)(3)]$$

$$= 3.5[10 + 18]$$

$$= 3.5(28)$$

$$S_7 = 98$$

Example 2: An arithmetic series has: $S_{20} = \frac{430}{3}$, $d = \frac{1}{3}$, $t_{20} = \frac{31}{3}$. Determine the first 3 terms of this series.

t_1, t_2, t_3

$$S_n = \frac{n}{2}(t_1 + t_n)$$

$$S_{20} = \frac{20}{2}(t_1 + t_{20})$$

$$\frac{430}{3} = 10\left(t_1 + \frac{31}{3}\right)$$

$$\frac{430}{3} = 10t_1 + \frac{310}{3}$$

$$\frac{-310}{3} \qquad \qquad \frac{-310}{3}$$

$$\frac{120}{3} = 10t_1$$

$$\frac{40}{10} = \frac{10t_1}{10}$$

$$4 = t_1$$

$$t_2 = t_1 + d$$

$$= 4 + \frac{1}{3}$$

$$t_2 = 4\frac{1}{3} \quad \text{or} \quad \frac{13}{3}$$

$$t_3 = t_2 + d$$

$$= \frac{13}{3} + \frac{1}{3}$$

$$t_3 = \frac{14}{3}$$

Example 3: Determine the SUM of this arithmetic series: $5 + 8 + 11 + \dots + 53$.

$$t_1 = 5$$

$$d = 3$$

$$t_n = 53$$

need "n" first

$$t_n = t_1 + (n-1)d$$

$$53 = 5 + (n-1)(3)$$

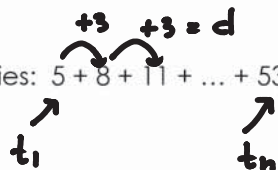
$$53 = 5 + 3n - 3$$

$$53 = 2 + 3n$$

$$\begin{array}{r} -2 \\ -2 \end{array}$$

$$\frac{51}{3} = \frac{3n}{3}$$

$$17 = n$$



$$S_n = \frac{n}{2}(t_1 + t_n)$$

$$S_{17} = \frac{17}{2}(5 + 53)$$

$$= 8.5(58)$$

$$S_n = 493$$

Example 4: Find the sum: $-3x + x + 5x + \dots + 65x$

$$t_1 = -3x$$

$$d = 4x$$

$$t_n = 65x$$

need "n" first

$$t_n = t_1 + (n-1)d$$

$$65x = -3x + (n-1)(4x)$$

$$65x = -3x + 4xn - 4x$$

$$65x = -7x + 4xn$$

$$\begin{array}{r} +7x \\ +7x \end{array}$$

$$\frac{72x}{4x} = \frac{4xn}{4x}$$

$$18 = n$$

$$S_n = \frac{n}{2}(t_1 + t_n)$$

$$S_{18} = \frac{18}{2}(-3x + 65x)$$

$$= 9(62x)$$

$$S_{18} = 558x$$

Practice: p.27 #1bc, 2acd, 3a, 4a, 5a, 6a, 7a, 8, 10, 11, 23a

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Pre-Calc 11