## 1.2 Arithmetic Series

An **Arithmetic Series** is the ___Sum___ of the terms in an Arithmetic Sequence.

$$S_n = \frac{n}{2}(t_1 + t_n)$$

$t_1$ = first term
$n$ = the number of terms
$t_n$ = the nth term (sum up to)
$S_n$ = sum of the first n terms

$$S_n = \frac{n}{2}[2t_1 + (n-1)d]$$

$t_1$ = first term
$d$ = common difference
$n$ = the number of terms
$S_n$ = sum of the first n terms

**Example 1**: What is the sum of the first 7 terms of the following arithmetic series: 5 + 8 + 11 + 14 +...?

$t_1 = 5$
$d = 3$
$n = 7$

$S_n = \frac{n}{2}(t_1 + t_n)$ or $S_n = \frac{n}{2}[2t_1 + (n-1)d]$

we don't know this term. We must use the other formula

$S_7 = \frac{7}{2}[2(5) + (7-1)(3)]$

$= 3.5[10 + 18]$

$= 3.5(28)$

$S_7 = 98$

**Example 2:** An arithmetic series has: $S_{20} = \frac{430}{3}$, $d = \frac{1}{3}$, $t_{20} = \frac{31}{3}$. Determine the first 3 terms of this series.

$t_1, t_2, t_3$

$S_n = \frac{n}{2}(t_1 + t_n)$

$S_{20} = \frac{20}{2}(t_1 + t_{20})$

$\frac{430}{3} = 10\left(t_1 + \frac{31}{3}\right)$

$\frac{430}{3} = 10t_1 + \frac{310}{3}$

$-\frac{310}{3}$ $-\frac{310}{3}$

$\frac{120}{3} = 10t_1$

$\frac{40}{10} = \frac{10t_1}{10}$

$4 = t_1$

$t_2 = t_1 + d$

$= 4 + \frac{1}{3}$

$t_2 = 4\frac{1}{3}$ or $\frac{13}{3}$

$t_3 = t_2 + d$

$= \frac{13}{3} + \frac{1}{3}$

$t_3 = \frac{14}{3}$

**Example 3:** Determine the SUM of this arithmetic series: $5 + 8 + 11 + \ldots + 53$.

+3 +3 = d

$t_1$ $t_n$

$t_1 = 5$

$d = 3$

$t_n = 53$

need "n" first

$t_n = t_1 + (n-1)d$

$53 = 5 + (n-1)(3)$

$53 = 5 + 3n - 3$

$53 = 2 + 3n$

$-2 \quad -2$

$\frac{51}{3} = \frac{3n}{3}$

$17 = n$

$S_n = \frac{n}{2}(t_1 + t_n)$

$S_{17} = \frac{17}{2}(5 + 53)$

$= 8.5(58)$

$S_{17} = 493$

**Example 4:** Find the sum: $-3x + x + 5x + \ldots + 65x$

$t_1 = -3x$

$d = 4x$

$t_n = 65x$

need "n" first

$t_n = t_1 + (n-1)d$

$65x = -3x + (n-1)(4x)$

$65x = -3x + 4xn - 4x$

$65x = -7x + 4xn$

$+7x \quad +7x$

$\frac{72x}{4x} = \frac{4xn}{4x}$

$18 = n$

$S_n = \frac{n}{2}(t_1 + t_n)$

$S_{18} = \frac{18}{2}(-3x + 65x)$

$= 9(62x)$

$S_{18} = 558x$

**Practice:** p.27 #1bc, 2acd, 3a, 4a, 5a, 6a, 7a, 8, 10, 11, 23a

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