1.3 Geometric Sequences (part 2)

Example 1: In nature, many single-celled organisms, such as bacteria, reproduce by splitting in two so that one cell gives rise to 2, then 4, then 8 cells, and so on, producing a geometric sequence. Suppose there were 10 bacteria originally present in a bacteria sample.

a) State the value of t_1 and r.

$t_1 = 10$ $\quad r = 2$ (splitting in two)

b) Determine the general term that relates the number of bacteria to the doubling period of the bacteric

$$t_n = t_1 \cdot r^{n-1}$$

$$t_n = 10 \cdot 2^{n-1}$$

0.06

Example 2: A car depreciates by 6% per year.

To find "r" 1) change % into a decimal

2) depreciation (% decrease)

$r = 1 - \_\_$

appreciation/growth (% increase)

$r = 1 + \_\_$

For this example: $r = 1 - 0.06 = 0.94$

a) How much is the car worth after 1 year, and after 10 years?

$t_n = t_1 \cdot r^{n-1}$

No initial value was given.
Assume that the car is worth 100% initially
So $t_1 = 100$ (this is at year zero)

After 1 year: $t_2 = t_1 \cdot r^{2-1}$

$n = 2$

$= (100)(0.94)^1$

$t_2 = 94$ } the car is worth 94% of its original value after 1 year

After 10 years: $t_{11} = t_1 \cdot r^{11-1}$

$n = 11$

$= (100)(0.94)^{10}$

$t_{11} = 53.9$ } car is worth 54% of its original value after 10 years

b) After how many years is the car worth 83% of its value?

↳ must find "n" first t_n

$t_n = t_1 \cdot r^{n-1}$

$\frac{83}{100} = \frac{100 \cdot (0.94)^{n-1}}{100}$

$0.83 = (0.94)^{n-1}$

we will guess & check values of "n"

try $n=5$: $(0.94)^{5-1} = (0.94)^4 = 0.7807$

$n=4$: $(0.94)^{4-1} = (0.94)^3 = 0.8306$ pretty close

so, $n=4$ (4 terms)

# of years = $n-1$

The car is worth 83% after 3 years.

Example 3: A ball is dropped from a height of 38.28 m. After each bounce, it rises to 60% of its previous height.

original height (no bounces)

a) What is the general term of this geometric sequence?

$t_1 = 38.28$

$r = 0.6$

$t_n = t_1 \cdot r^{n-1}$

$t_n = (38.28)(0.6)^{n-1}$

b) What height does the ball reach after the 7th bounce?

7 bounces ⟶ 8 terms ($n=8$)

$t_8 = t_1 \cdot r^{8-1}$

$= (38.28)(0.6)^7$

$= 1.07\,m$

c) After how many bounces will the ball reach a height of approximately 5 m?

↳ need n first t_n

$t_n = t_1 \cdot r^{n-1}$

$\frac{5}{38.28} = \frac{(38.28)(0.6)^{n-1}}{38.28}$

$0.1306 = 0.6^{n-1}$

guess & check values of n

$n=6$: $(0.6)^{6-1} = 0.6^5 = 0.07776$

$n=5$: $(0.6)^{5-1} = 0.6^4 = 0.1296$ pretty close

# of bounces $= n-1$

4 bounces

Practice: p.40 #8 – 10, 14, 18, 23