1.4 Geometric Series

A **Geometric Series** is the expression for the **sum** of a Geometric Sequence.

The general formula for the **sum**: $S_n = \frac{t_1(r^n - 1)}{r - 1}, r \neq 1$ or $S_n = \frac{rt_n - t_1}{r - 1}, r \neq 1$

Example 1: Find the sum of the first 15 terms: $40 - 20 + 10 - 5 + 2.5 - \cdots$

$n = 15$

$t_1 = 40$

$r = \frac{-20}{40} = -0.5$

$S_n = \frac{t_1(r^n - 1)}{r - 1}$

$S_{15} = \frac{40\left[(-0.5)^{15} - 1\right]}{-0.5 - 1}$

$= \frac{40(-1.00030518)}{-1.5}$

$S_{15} = 26.67$ approx.

Example 2: The first ticket of a draw wins a prize of \$10. Each ticket after that receives a prize that is 3 times the value of the preceding prize. Write the general term. Approximately how many prizes are given if the total amount of money given is \$ 2 700 000?

$t_1 = 10$

$r = 3$

$S_n = 2\,700\,000$ ← S_n

$n = ?$

general term: $t_n = t_1 \cdot r^{n-1}$

$t_n = 10 \cdot (3)^{n-1}$

$S_n = \frac{t_1(r^n - 1)}{r - 1}$

$2\,700\,000 = \frac{10(3^n - 1)}{3 - 1}$

$2\,700\,000 = \frac{10(3^n - 1)}{2}$ (5)

$\frac{2\,700\,000}{5} = \frac{5(3^n - 1)}{5}$

$540\,000 = 3^n - 1$

$+1 \quad +1$

$540\,001 = 3^n$ guess & check "n"

try $n = 11 : 3^{11} = 177147$

$n = 12 : 3^{12} = 531441$ } closest value

Approx 12 prizes given

Example 3: Find the SUM of the geometric series: $\frac{1}{27}+\frac{1}{9}+\frac{1}{3}+\ldots+729$

$t_1 = \frac{1}{27}$

$t_n = 729$

$r = \frac{(1/9)}{(1/27)} = 3$

$S_n = \frac{r \cdot t_n - t_1}{r-1}$

$= \frac{(3)(729) - (1/27)}{3-1}$

$= \frac{2187 - (1/27)}{2}$

$= 1093.5$ (approx)

Example 4: Find t_1 if $S_n = 765$, $t_n = 384$, and $r = 2$

$S_n = \frac{r \cdot t_n - t_1}{r-1}$

$765 = \frac{(2)(384) - t_1}{2-1}$

$765 = 768 - t_1$

$-768 \quad -768$

$\frac{-3}{-1} = \frac{-t_1}{-1}$

$3 = t_1$

Practice: p.53 #2ad, 3ac, 4bc, 5ab, 6, 9, 11, 16, 17, 19

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Pre-Calc 11