1.5 Infinite Geometric Series

An **Infinite Geometric Series** is a geometric series that does not end or it has no last term.

The sum of an infinite geometric series can be found if: $-1 < r < 1$ using for formula:

$$S_\infty = \frac{t_1}{1-r} \text{ where } r \neq 1$$

proper fraction

If the sum exists (meaning $-1 < r < 1$), then we can say that the geometric series is convergent.
The sum approaches a fixed value.

r is an improper fraction or whole number

If the sum does not exist (meaning $r > 1$ or $r < -1$), then we can say that the geometric series is divergent. The sum does not approach a fixed value.

Example 1: Decide whether each infinite geometric series is convergent or divergent. Find the sum of the series, if the sum exists.

a) $1-\frac{1}{3}+\frac{1}{9}-...$

$r = \frac{(-1/3)}{1} = -\frac{1}{3}$ convergent (the sum exists)

$S_\infty = \frac{t_1}{1-r} = \frac{1}{1-(-1/3)} = \frac{1}{4/3}$

$S_\infty = \frac{3}{4}$

b) $2+4+8+...$

$r = \frac{4}{2} = 2$ divergent ; $r > 1$ (the sum does not exist)

c) $4-12+36-108+...$

$r = \frac{-12}{4} = -3$ divergent ; $r < -1$ (the sum does not exists)

d) $4+2+1+0.5+...$

$r = \frac{2}{4} = 0.5$ convergent (the sum exists)

$S_\infty = \frac{t_1}{1-r} = \frac{4}{1-0.5} = \frac{4}{0.5}$

$S_\infty = 8$

Example 2: The first term of an infinite geometric series is 6 and its sum is $\frac{21}{4}$.

a) What is the common ratio?

$t_1 = 6$

$S_\infty = \frac{21}{4}$

$r = ?$

$S_\infty = \frac{t_1}{1-r}$

$\frac{21}{4} = \frac{6}{1-r}$

$(21)(1-r) = (6)(4)$

$21 - 21r = 24$

$-21 \qquad -21$

$\frac{-21r}{-21} = \frac{3}{-21}$

$r = -\frac{1}{7}$

b) Write the first four terms of the series.

$t_1 = 6$

$t_2 = 6 \cdot \left(-\frac{1}{7}\right) = -\frac{6}{7}$

$t_3 = -\frac{6}{7} \cdot \left(-\frac{1}{7}\right) = \frac{6}{49}$

$t_4 = \frac{6}{49} \cdot \left(-\frac{1}{7}\right) = -\frac{6}{343}$

$6 - \frac{6}{7} + \frac{6}{49} - \frac{6}{343} + \ldots$

1, 2, 5a, 6, 7, 8a, 12

Practice: p.63 #~~1, 2abcd, 3ab, 6, 8a,~~ 12