4.4 Fractional Exponents and Radicals

Use a calculator to complete the following tables. (Use the buttons x^n or $\sqrt[x]{\ }$)

x	$x^{\frac{1}{2}}$
1	1
4	2
9	3
16	4
25	5
36	6

x	$x^{\frac{1}{3}}$
1	1
8	2
27	3
64	4
125	5
216	6

Notice a pattern :

$\sqrt{x}$ the square root in exponential form is: $x^{\frac{1}{2}}$

$\sqrt[3]{x}$ the cube root in exponential form is : $x^{\frac{1}{3}}$

We can keep going..... $\sqrt[5]{x}$ the fifth root in exponential for mis : $x^{\frac{1}{5}}$

denominator is the index

In general, $x^{\frac{1}{n}}$ as a radical becomes $\sqrt[n]{x}$. n And vice versa, $\sqrt[n]{x}$ equals $x^{\frac{1}{n}}$.

Example 1: Write as a radical and then evaluate.

a) $1000^{\frac{1}{3}}$

$= \sqrt[3]{1000}$

$= 10$

b) $0.25^{\frac{1}{2}}$

$= \sqrt{0.25}$

$= 0.5$

c) $27^{\frac{1}{3}}$

$\sqrt[3]{27}$

$= 3$

d) $\left(\frac{16}{81}\right)^{\frac{1}{4}}$

$\sqrt[4]{\frac{16}{81}}$

$= \frac{2}{3}$

or

$\frac{\sqrt[4]{16}}{\sqrt[4]{81}}$

$= \frac{2}{3}$

e) $\left(\frac{4}{9}\right)^{\frac{1}{2}}$

$\sqrt{\frac{4}{9}}$ or $\frac{\sqrt{4}}{\sqrt{9}}$

$= \frac{2}{3}$

f) $(-64)^{\frac{1}{3}}$

$\sqrt[3]{-64}$

$= -4$

Powers with Rational Exponents

When m and n are natural numbers, and x is a rational number:

$$x^{\frac{m}{n}} = \left(\sqrt[n]{x}\right)^m \quad \text{or} \quad x^{\frac{m}{n}} = \sqrt[n]{x^m}$$

preferred

Example 2: Write $26^{\frac{2}{5}}$ in radical form in two different ways.

i) $26^{\frac{2}{5}} = \left(\sqrt[5]{26}\right)^2$ ii) $26^{\frac{2}{5}} = \sqrt[5]{26^2}$

Example 3: Write in exponential form.

a) $\sqrt{3^5}$ $\quad 3^{\frac{5}{2}}$

b) $\left(\sqrt[3]{25}\right)^2$ $\quad 25^{\frac{2}{3}}$

Example 4: Write in radical form and then evaluate.

a) $8^{\frac{2}{3}}$

$\left(\sqrt[3]{8}\right)^2 = 2^2 = 4$ or $\sqrt[3]{8^2} = \sqrt[3]{64} = 4$

b) $81^{\frac{3}{4}}$

$\left(\sqrt[4]{81}\right)^3 = 3^3 = 27$ or $\sqrt[4]{81^3} = \sqrt[4]{531,441} = 27$

c) $(-32)^{0.4}$ convert to fraction first $0.4 = \frac{4}{10} = \frac{2}{5}$

$(-32)^{\frac{2}{5}} = \left(\sqrt[5]{(-32)}\right)^2 = (-2)^2 = 4$

d) $0.04^{\frac{3}{2}}$

e) $27^{\frac{4}{3}}$

f) ~~$\frac{1}{2}$~~

Practice: p. 227 # 3 – 8, 10 – 12, 16

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