

## Chapter 3 Final Review

1. Factor each polynomial.

a)  $2a^3b - 8a^2b$

$$= 2a^2b(a-4)$$

b)  $9x^3y^4 + 27xy^2$

$$= 9xy^2(x^2y^2 + 3)$$

c)  $-3xy^6 - 15x^2y^2 + 3xy$

$$= -3xy(y^5 + 5xy - 1)$$

d)  $2y^2x^5 + 2y^6 + 4y$

$$= 2y(yx^5 + y^5 + 2)$$

e)  $72x^2y^3 - 48x^3y + 80x^2y$

$$= 8x^2y(9y^2 - 6x + 10)$$

f)  $18u^7v^5 - 54u^3v^3 - 27uv^2$

$$= 9uv^2(2u^6v^3 - 6u^2v - 3)$$

g)  $-9a^4b^5 - 12a^2b^3 + 6a^3$

$$= -3a^2(3a^2b^5 + 4b^3 - 2a)$$

h)  $24y^6 + 20y^3 - 8y^2x$

$$= 4y^2(6y^4 + 5y - 2x)$$

i)  $-12f^8g^6 + 6f^6g^7 + 12f^7g^5 + 54f^6g^3$

$$-6f^6g^3(2f^2g^3 - g^4 - 2fg^2 - 9)$$

$\uparrow$   
nine,  
not "g"

j)  $-21ab^3 - 21a^2b^2 - 35ab^2 - 14a^2b$

$$= -7ab(3b^2 + 3ab + 5b - 2a)$$

2. Factor each trinomial.

a)  $x^2 - 9x + 18$      $\underline{-6} \times \underline{-3} = 18$   
 $\underline{-6} + \underline{-3} = -9$

$$= (x-6)(x-3)$$

b)  $x^2 - 15x + 54$      $\underline{-9} \times \underline{-6} = 54$   
 $\underline{-9} + \underline{-6} = -15$

$$= (x-9)(x-6)$$

c)  $p^2 - 14p + 40$      $\underline{-4} \times \underline{-10} = 40$   
 $\underline{-4} + \underline{-10} = -14$

$$= (p-4)(p-10)$$

d)  $x^2 + x - 42$      $\underline{1} \times \underline{-6} = -42$   
 $\underline{1} + \underline{-6} = 1$

$$= (x+7)(x-6)$$

$$\text{e) } n^2 - 12n + 35 \quad \underline{-7} \times \underline{-5} = 35 \\ \underline{-7} + \underline{-5} = -12 \\ = (n-7)(n-5)$$

$$\text{f) } n^2 + 9n + 14 \quad \underline{1} \times \underline{2} = 14 \\ \underline{1} + \underline{2} = 9 \\ = (n+7)(n+2)$$

$$\text{g) } r^2 + 3r - 28 \quad \underline{1} \times \underline{-4} = -28 \\ \underline{1} + \underline{-4} = 3 \\ = (r+7)(r-4)$$

$$\text{h) } x^2 + 7x + 6 \quad \underline{1} \times \underline{6} = 6 \\ \underline{1} + \underline{6} = 7 \\ = (x+1)(x+6)$$

$$\text{i) } r^2 - r - 12 \quad \underline{-4} \times \underline{3} = -12 \\ \underline{-4} + \underline{3} = -1 \\ = (r-4)(r+3)$$

$$\text{j) } m^2 + 5m - 50 \quad \underline{10} \times \underline{-5} = -50 \\ \underline{10} + \underline{-5} = 5 \\ = (m+10)(m-5)$$

3. Factor each trinomial.

$$\text{a) } \underbrace{5r^2 + r - 4}_{(5r-4) = -20} \quad \underline{5} \times \underline{-4} = -20 \\ \underline{5} + \underline{-4} = 1$$

$$5r^2 + 5r \cancel{-} 4r - 4$$

$$5r(r+1) - 4(r+1)$$

$$= (r+1)(5r-4)$$

$$\text{b) } \underbrace{3k^2 - 7k - 10}_{(3k-10) = -30} \quad \underline{-10} \times \underline{3} = -30 \\ \underline{-10} + \underline{3} = -7$$

$$3k^2 + 3k \cancel{-} 10k - 10$$

$$3k(k+1) - 10(k+1)$$

$$= (k+1)(3k-10)$$

$$\text{c) } \underbrace{7x^2 + 20x + 12}_{(7x+12) = 84} \quad \underline{6} \times \underline{14} = 84 \\ \underline{6} + \underline{14} = 20$$

$$7x^2 + 14x \cancel{+} 6x + 12$$

$$7x(x+2) + 6(x+2)$$

$$= (x+2)(7x+6)$$

$$\text{d) } \underbrace{3x^2 - 2x - 16}_{(3x-16) = -48} \quad \underline{-8} \times \underline{6} = -48 \\ \underline{-8} + \underline{6} = -2$$

$$3x^2 + 6x \cancel{-} 8x - 16$$

$$3x(x+2) - 8(x+2)$$

$$= (x+2)(3x-8)$$

$$\text{e) } \underbrace{7p^2 - 5p - 2}_{(7)(-2) = -14} \quad \underline{-7} \times \underline{2} = -14 \\ \underline{-7} + \underline{2} = -5$$

$$7p^2 - 7p \cancel{+} 2p - 2$$

$$7p(p-1) + 2(p-1)$$

$$= (p-1)(7p+2)$$

$$\text{f) } \underbrace{9x^2 - 12x - 32}_{(9)(-32) = -288} \quad \underline{-24} \times \underline{12} = -288 \\ \underline{-24} + \underline{12} = -12$$

$$9x^2 - 24x \cancel{+} 12x - 32$$

$$3x(3x-8) + 4(3x-8)$$

$$= (3x-8)(3x+4)$$

$$g) \begin{array}{l} \overbrace{4r^2 + 15r - 25}^{\frac{20}{(4r-25)} = -100} \\ (4r-25) = -100 \end{array}$$

$$\begin{array}{l} \frac{20}{20} \times \frac{-5}{-5} = -100 \\ \underline{20} + \underline{-5} = 15 \end{array}$$

$$h) \begin{array}{l} \overbrace{4x^2 + x - 5}^{\frac{-4}{(4x-5)} = -20} \\ (4x-5) = -20 \end{array}$$

$$\begin{array}{l} \frac{-4}{-4} \times \frac{5}{5} = -20 \\ \underline{-4} + \underline{5} = 1 \end{array}$$

$$4r^2 + 20r - 5r - 25$$

$$4r(r+5) - 5(r+5)$$

$$= (r+5)(4r-5)$$

$$4x^2 - 4x + 5x - 5$$

$$4x(x-1) + 5(x-1)$$

$$= (x-1)(4x+5)$$

$$i) \begin{array}{l} \overbrace{9p^2 + 52p - 12}^{\frac{54}{(9)(-12)} = -108} \\ (9)(-12) = -108 \end{array}$$

$$\begin{array}{l} \frac{54}{54} \times \frac{-2}{-2} = -108 \\ \underline{54} + \underline{-2} = 52 \end{array}$$

$$j) \begin{array}{l} \overbrace{9x^2 + 7x - 2}^{\frac{9}{(9)(-2)} = -18} \\ (9)(-2) = -18 \end{array}$$

$$\begin{array}{l} \frac{9}{9} \times \frac{-2}{-2} = -18 \\ \underline{9} + \underline{-2} = 7 \end{array}$$

$$9p^2 + 54p - 2p - 12$$

$$9p(p+6) - 2(p+6)$$

$$= (p+6)(9p-2)$$

$$9x^2 + 9x - 2x - 2$$

$$9x(x+1) - 2(x+1)$$

$$= (x+1)(9x-2)$$

4. Factor each polynomial.

$$a) 16x^2 - 9y^2$$

$$= (4x-3y)(4x+3y)$$

$$b) 49x^2 - 25$$

$$= (7x-5)(7x+5)$$

$$c) 4a^2 - 9b^2$$

$$= (2a-3b)(2a+3b)$$

$$d) 225 - 121x^2$$

$$= (15-11x)(15+11x)$$

$$e) 4x^2 + 12x + 9$$

$$= (2x+3)(2x+3)$$

$$= (2x+3)^2$$

$$f) 16x^2 + 24x + 9$$

$$= (4x+3)(4x+3)$$

$$= (4x+3)^2$$

$$g) 25x^2 - 20x + 4$$

$$= (5x-2)(5x-2)$$

$$= (5x-2)^2$$

$$h) 9x^2 - 30x + 25$$

$$= (3x-5)(3x-5)$$

$$= (3x-5)^2$$

$$i) 25x^2 - 10x + 1$$

$$= (5x-1)(5x-1)$$

$$= (5x-1)^2$$

$$j) a^2 + 8a + 16$$

$$= (a+4)(a+4)$$

$$= (a+4)^2$$

5. Expand and simplify each expression.

a)  $(x+3)(x+2)$

$$= x^2 + 2x + 3x + 6$$

$$= x^2 + 5x + 6$$

c)  $(3a+2)(a-5)$

$$= 3a^2 - 15a + 2a - 10$$

$$= 3a^2 - 13a - 10$$

e)  $(2p+3)(3p-1)$

$$= 6p^2 - 2p + 9p - 3$$

$$= 6p^2 + 7p - 3$$

g)  $(2b+7)(2b^2 - 2b - 4)$

$$= 4b^3 - 4b^2 - 8b + 14b^2 - 14b - 28$$

$$= 4b^3 + 10b^2 - 22b - 28$$

i)  $(3x^2 + 6x + 8)(8x - 5)$

$$= 24x^3 - 15x^2 + 48x^2 - 30x + 64x - 40$$

$$= 24x^3 + 33x^2 + 34x - 40$$

k)  $2(x+3) + 3(3x-2)$

$$= 2x + 6 + 9x - 6$$

$$= 11x$$

m)  $2x(x-3) + x - 4(2x+5)$

$$= 2x^2 - 6x + x - 8x - 20$$

$$= 2x^2 - 13x - 20$$

b)  $(x-4)(x+8)$

$$= x^2 + 8x - 4x - 32$$

$$= x^2 + 4x - 32$$

d)  $(2x-5)(4x-7)$

$$= 8x^2 - 14x - 20x + 35$$

$$= 8x^2 - 34x + 35$$

f)  $(4x-1)(7x-9)$

$$= 28x^2 - 36x - 7x + 9$$

$$= 28x^2 - 43x + 9$$

h)  $(4n+1)(4n^2 + 3n + 8)$

$$= 16n^3 + 12n^2 + 32n + 4n^2 + 3n + 8$$

$$= 16n^3 + 16n^2 + 35n + 8$$

j)  $(5x^2 - 6x + 1)(7x - 7)$

$$= 35x^3 - 35x^2 - 42x^2 + 42x + 7x - 7$$

$$= 35x^3 - 77x^2 + 49x - 7$$

l)  $(x+2)^2 - 4(x+1)$

$$= (x+2)(x+2) - 4(x+1)$$

$$= x^2 + 2x + 2x + 4 - 4x - 4$$

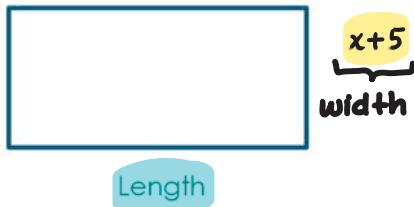
$$= x^2 + 4x + 4 - 4x - 4 = x^2$$

n)  $2 + 4(x^2 - 3x + 4) - (x+7)$

$$= 2 + 4x^2 - 12x + 16 - x - 7$$

$$= 4x^2 - 13x + 11$$

6. Given that the area of the rectangle below is  $\underline{\underline{2x^2 + 9x - 5}}$ , determine the length of the rectangle.



$$\text{Area} = 2x^2 + 9x - 5$$

$$\text{Area} = \text{Length} \times \text{width}$$

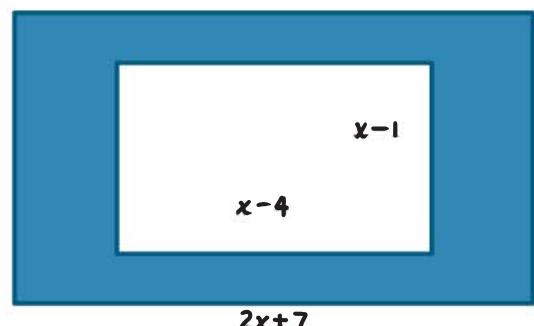
$$\begin{array}{rcl} 2x^2 + 9x - 5 & & \frac{10}{10} \times \frac{-1}{-1} = -10 \\ \cancel{(2x-5)} & & \cancel{+} \frac{-1}{-1} = 9 \end{array}$$

$$2x^2 + 10x - 1x - 5$$

$$2x(x+5) - 1(x+5)$$

$$(x+5)(2x-1) \quad \leftarrow \text{length}$$

7. Determine an expression to represent the shaded area below.



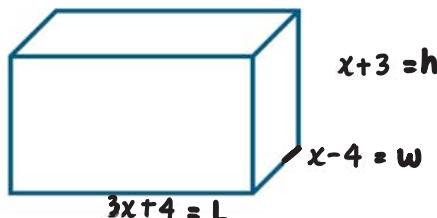
$$\text{Shaded area} = \text{large rectangle} - \text{small rectangle}$$

$$\begin{array}{r} 2x^2 + 17x + 35 \\ - (x^2 - 5x + 4) \\ \hline x^2 + 22x + 31 \end{array}$$

Area of small rectangle :  $(x-4)(x-1)$   
 $= x^2 - x - 4x + 4$   
 $= x^2 - 5x + 4$

Area of large rectangle :  $(2x+7)(x+5)$   
 $= 2x^2 + 10x + 7x + 35$   
 $= 2x^2 + 17x + 35$

8. Determine a simplified expression for the volume of the prism below. Remember,  $V = L \times w \times h$ .



$$\begin{aligned} V &= (3x+4)(x-4)(x+3) \\ &= (3x^2 - 12x + 4x - 16)(x+3) \\ &= (3x^2 - 8x - 16)(x+3) \end{aligned}$$

$$\begin{aligned} V &= (3x^2 - 8x - 16)(x+3) \\ &= 3x^3 + 9x^2 - 8x^2 - 24x - 16x - 48 \\ &= 3x^3 + x^2 - 40x - 48 \end{aligned}$$

Additional Practice: p.199 #11bdf, 12abc, 13, 18acef, 19, 24bdf, 25, 27ac, 30, 32, 33ace