Chapter 3 Final Review

A. Factoring

1) Greatest Common Factor (GCF)

- Look at each term of the polynomial and determine what they have in common.
  - Each coefficient must be divisible by the same number
  - Variable(s) must have the same minimum exponent in common

Example: Factor

a) $\dfrac{5x^2yz^4}{5xyz^3} - \dfrac{15xy^2z^3}{5xyz^3}$ GCF = $5xyz^3$

$= 5xyz^3(xz - 3y)$

b) $\dfrac{-24ab^5}{-6b^3} + \dfrac{30b^4c^2}{-6b^3} + \dfrac{12ab^3c}{-6b^3}$ GCF = $-6b^3$

$= -6b^3(4ab^2 - 5bc^2 - 2ac)$

2) Factoring Trinomials

***Simple Trinomials:** $x^2 + bx + c$

- Determine two integers whose **product** is c and whose **sum** is b
- Write the variable and each integer in a set of brackets

$\left.\begin{array}{l} \_\_ \times \_\_ = c \\ \_\_ + \_\_ = b \end{array}\right.$

$(x \quad)(x \quad)$

Example: Factor

a) $x^2 - 5x - 6$

$\underline{-6} \times \underline{1} = -6$

$\underline{-6} + \underline{1} = -5$

multiply to a negative value so one number must be negative

$(x-6)(x+1)$

b) $x^2 - 7x + 12$

$\underline{-4} \times \underline{-3} = 12$

$\underline{-4} + \underline{-3} = -7$

multiply to a positive value so both numbers must be negative.

$(x-4)(x-3)$

*Complex Trinomials ax^2+bx+c ac = magic number

- Determine two integers whose **product** is ac (our magic number) and whose **sum** is b
- Factor by **decomposition**

$\_\_ \times \_\_ = ac$

$\_\_ + \_\_ = b$

Example: Factor

a) $6x^2+17x+5$

$(6)(5) = 30$

$15 \times 2 = 30$

$15 + 2 = 17$

$6x^2 + 15x + 2x + 5$

$3x(2x+5) + 1(2x+5)$

$(2x+5)(3x+1)$

decomposition

b) $3x^2+x-4$

$(3)(-4) = -12$

$4 \times -3 = -12$

$4 + -3 = 1$

$3x^2 - 3x + 4x - 4$

$3x(x-1) + 4(x-1)$

$(x-1)(3x+4)$

3) Factoring Perfect Square Trinomials ax^2+bx+c

Perfect square trinomials have the following characteristics:

- a and c must be perfect squares (1, 4, 9, 16, etc….)
- bx is equal to twice the product of the square roots of terms a and c.

Example: Factor

a) $9x^2+12x+4$

$\sqrt{9x^2} = 3x$ $\quad \sqrt{4} = 2$

$= (3x+2)(3x+2)$

$= (3x+2)^2$

b) $4x^2-20x+25$

$\sqrt{4x^2} = 2x$ $\quad \sqrt{25} = 5$

$= (2x-5)(2x-5)$

$= (2x-5)^2$

4) Factoring Difference of Squares a^2-b^2

The difference of squares has the following characteristics:

- There are only two terms in the polynomial (binomial)
- Each term is a perfect square
- Second term must be subtracted from the first term

Example: Factor

a) x^2-144

$\sqrt{x^2} = x$ $\quad \sqrt{144} = 12$

$= (x-12)(x+12)$

b) $4x^2-49$

$\sqrt{4x^2} = 2x$ $\quad \sqrt{49} = 7$

$= (2x-7)(2x+7)$

5) Multiplying Polynomials

Use the **DISTRIBUTIVE PROPERTY** to multiply polynomials

- Multiply each term of the first polynomial with each term of the second polynomial.
- Collect like terms and simplify.

Example: Expand and Simplify

a) $(x-3)(x+4)$

$= (x)(x) + (x)(4) + (-3)(x) + (-3)(4)$

$= x^2 + 4x - 3x - 12$

$= x^2 + x - 12$

b) $4x^2(x^2+3x-2)$

$= (4x^2)(x^2) + (4x^2)(3x) + (4x^2)(-2)$

$= 4x^4 + 12x^3 - 8x^2$

c) $3(x-2) - 7x(x-1)$

$= 3x - 6 - 7x^2 + 7x$

$= 10x - 6 - 7x^2$

d) $x - 2(x^2 - 9x - 3)$

$= x - 2x^2 + 18x + 6$

$= 19x - 2x^2 + 6$