

Chapter 3 Final Review

A. Factoring

1) Greatest Common Factor (GCF)

- Look at each term of the polynomial and determine what they have in common.
 - Each coefficient must be divisible by the same number
 - Variable(s) must have the same minimum exponent in common

Example: Factor

$$a) \frac{5x^2yz^4}{5xyz^3} - \frac{15xy^2z^3}{5xyz^3} \quad \text{GCF} = 5xyz^3$$

$$= 5xyz^3(xz - 3y)$$

$$b) \frac{-24ab^5}{-6b^3} + \frac{30b^4c^2}{-6b^3} + \frac{12ab^3c}{-6b^3} \quad \text{GCF} = -6b^3$$

$$= -6b^3(4ab^2 - 5bc^2 - 2ac)$$

2) Factoring Trinomials

*Simple Trinomials: $x^2 + bx + c$

- Determine two integers whose **product** is c and whose **sum** is b

$$\left. \begin{array}{l} \underline{\quad} \times \underline{\quad} = c \\ \underline{\quad} + \underline{\quad} = b \end{array} \right\}$$
- Write the variable and each integer in a set of brackets

$$(x \quad)(x \quad)$$

Example: Factor

$$a) x^2 - 5x - 6$$

$$\begin{array}{rcl} \underline{-6} & \times & \underline{1} = -6 \\ \underline{-6} & + & \underline{1} = -5 \end{array}$$

$$(x-6)(x+1)$$

multiply to a negative value
so one number must be negative

$$b) x^2 - 7x + 12$$

$$\begin{array}{rcl} \underline{-4} & \times & \underline{-3} = 12 \\ \underline{-4} & + & \underline{-3} = -7 \end{array}$$

$$(x-4)(x-3)$$

multiply to a positive value
so both numbers must be negative.

*Complex Trinomials $ax^2 + bx + c$ $ac = \text{magic number}$

- Determine two integers whose **product** is ac (our magic number) and whose **sum** is b
- Factor by **decomposition**

$$\begin{array}{rcl} _ & \times & _ = ac \\ _ & + & _ = b \end{array}$$

Example: Factor

a) $6x^2 + 17x + 5$

$$\begin{array}{rcl} \uparrow & & \uparrow \\ (6)(5) = 30 & & \end{array}$$

$$\begin{array}{rcl} 15 & \times & 2 = 30 \\ 15 & + & 2 = 17 \end{array}$$

b) $3x^2 + x - 4$

$$\begin{array}{rcl} \uparrow & & \uparrow \\ (3)(-4) = -12 & & \end{array}$$

$$\begin{array}{rcl} 4 & \times & -3 = -12 \\ 4 & + & -3 = 1 \end{array}$$

$$\begin{array}{c} 6x^2 + 15x + 2x + 5 \\ 3x(2x+5) + 1(2x+5) \\ (2x+5)(3x+1) \end{array} \quad \left. \vphantom{\begin{array}{c} 6x^2 + 15x + 2x + 5 \\ 3x(2x+5) + 1(2x+5) \\ (2x+5)(3x+1) \end{array}} \right\} \text{decomposition}$$

$$\begin{array}{c} 3x^2 - 3x + 4x - 4 \\ 3x(x-1) + 4(x-1) \\ (x-1)(3x+4) \end{array}$$

3) Factoring Perfect Square Trinomials $ax^2 + bx + c$

Perfect square trinomials have the following characteristics:

- a and c must be perfect squares (1, 4, 9, 16, etc....)
- bx is equal to twice the product of the square roots of terms a and c .

Example: Factor

a) $9x^2 + 12x + 4$

$$\begin{array}{l} \sqrt{9x^2} = 3x \quad \sqrt{4} = 2 \\ = (3x+2)(3x+2) \\ = (3x+2)^2 \end{array}$$

b) $4x^2 - 20x + 25$

$$\begin{array}{l} \sqrt{4x^2} = 2x \quad \sqrt{25} = 5 \\ = (2x-5)(2x-5) \\ = (2x-5)^2 \end{array}$$

4) Factoring Difference of Squares $a^2 - b^2$

The difference of squares has the following characteristics:

- There are only two terms in the polynomial (binomial)
- Each term is a perfect square
- Second term must be subtracted from the first term

Example: Factor

a) $x^2 - 144$

$$\begin{array}{l} \sqrt{x^2} = x \quad \sqrt{144} = 12 \\ = (x-12)(x+12) \end{array}$$

b) $4x^2 - 49$

$$\begin{array}{l} \sqrt{4x^2} = 2x \quad \sqrt{49} = 7 \\ = (2x-7)(2x+7) \end{array}$$

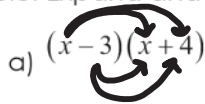
5) Multiplying Polynomials

Use the **DISTRIBUTIVE PROPERTY** to multiply polynomials

- Multiply each term of the first polynomial with each term of the second polynomial.
- Collect like terms and simplify.

Example: Expand and Simplify

a) $(x-3)(x+4)$

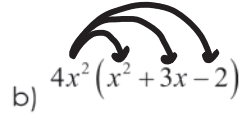


$$= (x)(x) + (x)(4) + (-3)(x) + (-3)(4)$$

$$= x^2 + 4x - 3x - 12$$

$$= x^2 + x - 12$$

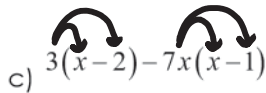
b) $4x^2(x^2 + 3x - 2)$



$$= (4x^2)(x^2) + (4x^2)(3x) + (4x^2)(-2)$$

$$= 4x^4 + 12x^3 - 8x^2$$

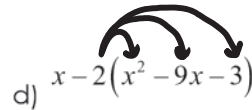
c) $3(x-2) - 7x(x-1)$



$$= 3x - 6 - 7x^2 + 7x$$

$$= 10x - 6 - 7x^2$$

d) $x-2(x^2-9x-3)$



$$= x - 2x^2 + 18x + 6$$

$$= 19x - 2x^2 + 6$$