## Chapter 4 Final Review

1. Write each mixed radical as an entire radical.
a) $3 \sqrt{8}$
b) $3 \sqrt[3]{4}$
$=\sqrt[3]{3^{3} \cdot 4}$
c) $6 \sqrt{9}$
$=\sqrt{3^{2}-8}$
$=\sqrt[3]{27.4}$
$=\sqrt[3]{108}$
$=\sqrt{6^{2} \cdot 9}$
$=\sqrt{9.8}$
$=\sqrt{72}$
e) $6 \sqrt[4]{3}$
$=\sqrt[4]{6^{4} \cdot 3}$
$=\sqrt[4]{1296 \cdot 3}$
$=\sqrt[4]{3888}$
$=\sqrt{36 \cdot 9}$
$=\sqrt{324}$
d) $-2 \sqrt[3]{5}$
$=\sqrt[3]{(-2)^{3} \cdot 5}$
$=\sqrt[3]{-8 \cdot 5}$
$=\sqrt[3]{-40}$
h) $7 \sqrt{6}$
$=\sqrt{7^{2} \cdot 6}$
$=\sqrt{49.6}$
$=\sqrt{294}$
2
f) $12 \sqrt{5 x}$
$=\sqrt{12^{2} \cdot 5} x$
$=\sqrt{144.5 x}$
$=\sqrt{720 x}$
g) $5 \sqrt[3]{3}$
$=\sqrt[3]{5^{3} \cdot 3}$
$=\sqrt[3]{125 \cdot 3}$
$=\sqrt[3]{375}$
i) $15 \sqrt{20}$
$=\sqrt{15^{2} \cdot 20}$
$=\sqrt{225 \cdot 20}$
$=\sqrt{4500}$
2. Write each radical in simplest form (mixed radical).
a) $\sqrt{20}$
$=\sqrt{4.5}$
$=2 \sqrt{5}$
b) $\sqrt{48}$
$=\sqrt{16 \cdot 3}$
c) $\sqrt{54}$
$=\sqrt{9.6}$
$=4 \sqrt{3}$
$=3 \sqrt{6}$
d) $\sqrt{72}$
e) $\sqrt{192}$
$=\sqrt{36 \cdot 2}$
$=\sqrt{64 \cdot 3}$
f) $\sqrt{45}$
$=8 \sqrt{3}$
$=\sqrt{9.5}$
$=6 \sqrt{2}$
h) $\sqrt{128}$
i) $\sqrt{175}$
g) $\sqrt{32}$
$=\sqrt{64.2}$
$=3 \sqrt{5}$
$=\sqrt{16 \cdot 2}$
$=8 \sqrt{2}$
$=\sqrt{25 \cdot 7}$
$=4 \sqrt{2}$
1) $\sqrt{216}$
k) $\sqrt{405}$
$=\sqrt{81.5}$
$=9 \sqrt{5}$
j) $\sqrt{180}$
$=\sqrt{36-5}$
$=6 \sqrt{5}$
$=5 \sqrt{7}$
3. Write each power as a radical.

$$
\begin{aligned}
& \text { a) } 4^{\frac{1}{4}} \\
& \quad=(\sqrt[4]{42})^{5} \text { or } \sqrt[4]{(42)^{5}}
\end{aligned}
$$

b) $\left(\frac{3}{4}\right)^{\frac{5}{6}}$
d) $\begin{aligned} &-64^{\frac{1}{3}} \\ &=\sqrt[3]{-64}\end{aligned}$

f) ${ }^{16^{\frac{2}{3}}}=(\sqrt[3]{16})_{\text {or }}^{2} \sqrt[3]{(16)^{2}}$
4. Write each power as a radical and evaluate.
a) $4^{\frac{1}{2}}=(\sqrt{4})^{5}$
b) $27^{\frac{1}{5}}=(\sqrt[3]{27})^{4}$
c) $81^{\frac{1}{4}}=(\sqrt[4]{81})^{3}$
$=3^{4}$
$=3^{3}$
$=2^{5}$
$=32$
$=81$
$=27$
d) $64^{\frac{3}{3}}=(\sqrt[3]{64})^{2}$
e) $25^{\frac{1}{2}}=(\sqrt{25})^{3}$
f) $1024^{\frac{1}{10}}=(\sqrt[n]{1024})^{3}$
$=5^{3}$
$=125$
5. Write each radical as a power.
a) $\sqrt{10^{3}}=10^{\frac{3}{2}}$
b) $(\sqrt[4]{6})^{3}=6^{\frac{3}{4}}$
c) ${ }^{(\sqrt[5]{62})^{3}}=62^{\frac{3}{5}}$
d) $\begin{aligned} & \sqrt{\left(\frac{3}{4}\right)^{9}} \\ & =\left(\frac{3}{4}\right)^{\frac{9}{2}}\end{aligned}$
e)
$\left(\sqrt[4]{\frac{1}{3}}\right)^{5}$
$=\left(\frac{1}{3}\right)^{\frac{5}{4}}$
f) ${ }^{5} \sqrt{\frac{2}{9}}$
$=\left(\frac{2}{9}\right)^{\frac{1}{5}}$
6. Write each power with a positive exponent.
a) $\left(\frac{14}{5}\right)^{-\frac{4}{3}}$
b) $\left(\frac{1}{3}\right)^{-5}$
$=\left(\frac{5}{14}\right)^{\frac{4}{3}}$
$=\left(\frac{3}{1}\right)^{5}$ or $3^{5}$
c) $\frac{-3^{-6}}{1} \rightarrow$ or $-1.3^{-6}$
$=\frac{1}{-3^{6}}$ or $\frac{-1}{3^{6}}$
d) $\frac{14^{-3}}{1}$
$=\frac{1}{14^{3}}$
e) $\frac{15^{-\frac{1}{4}}}{1}$
$=\frac{1}{15^{\frac{1}{4}}}$
f) $\frac{8^{-\frac{5}{2}}}{1}$
$=\frac{1}{8^{5 / 2}}$
7. Write each power with a positive exponent and evaluate.
a) $\frac{3^{-4}}{1}$
b) $\frac{49^{-\frac{1}{2}}}{1}$
$=\frac{1}{49^{3 / 2}}$
$=\frac{1}{3^{4}}$
$=\frac{1}{(\sqrt{49})^{3}}$
C) $\left(\frac{4}{3}\right)^{-3}$
$=\left(\frac{3}{4}\right)^{3}$
$=\frac{1}{81}$
$=\frac{27}{64}$
d) $16^{-0.25} \rightarrow-0.25=\frac{-25}{100}=\frac{-1}{4}$

$$
=\frac{1}{7^{3}}=\frac{1}{343}
$$

$$
\begin{aligned}
& =16^{-\frac{1}{4}} \\
& =\frac{1}{16^{1 / 4}} \\
& =\frac{1}{\sqrt[4]{16}}=\frac{1}{2}
\end{aligned}
$$

c) $\left(\frac{1}{6}\right)^{-4}$
e) $49^{-\frac{1}{2}}$
$=\left(\frac{6}{1}\right)^{4}$
"b"
$=\frac{1296}{1}$
$=1296$
8. Simplify each question and express your answer with positive exponents.
a) $3 a^{2} \cdot a^{-5} \cdot a^{4}$
b) $\left(2 x^{2} \cdot 3 x^{-5}\right)^{3}$
$=3 a^{2+(-5)+4}$
$=\left(6 x^{2+(-5)}\right)^{3}$
$=3 a^{\prime}$
$=3 a$
$=\left(6 x^{-3}\right)^{3}$
$=6^{3} x^{(-3)(3)}$
$=216 x^{-9}$
$=\frac{216}{x^{9}}$

$$
\begin{aligned}
& \text { c) } \frac{12 a^{2}}{3 a^{-3}} \\
& =4 a^{2-(-3)} \\
& =4 a^{5} \\
& \text { e) } \frac{\left(-5 h^{2} k^{-3}\right)^{-2}}{1} \\
& =\frac{1}{\left(-5 h^{2} k^{-3}\right)^{2}} \\
& =\frac{1}{(-5)^{2} h^{2 \cdot 2} K^{-3.2}} \\
& =\frac{1}{25 h^{4} K^{-6}}=\frac{K^{6}}{25 h^{4}} \\
& \text { g) } 2 x^{4} y^{2} \cdot\left(3 x y^{4}\right)^{2} \\
& =2 x^{4} y^{2} \cdot 3^{2} x^{12} y^{4 \cdot 2} \\
& =2 x^{4} y^{2} \cdot 9 x^{2} y^{8} \\
& =18 x^{4+2} y^{2+8} \\
& =18 x^{6} y^{10} \\
& \text { i) }\left(9 m^{-4} n^{6}\right)^{-\frac{1}{3}}\left(m^{-4} n\right) \\
& =\frac{1}{\left(9 m^{-4} n^{6}\right)^{\frac{1}{3}}} \cdot\left(m^{-4} n\right) \\
& =\frac{m^{-4} n}{9^{4 / 3} m^{-4 \cdot \frac{1}{3}} n^{6 \cdot-3}} \\
& =\frac{m^{-4} n}{\sqrt[3]{9} m^{-4 / 3} n^{2}} \rightarrow-\frac{4^{x^{3}}-\left(-\frac{4}{3}\right)}{1_{83}} \\
& =\frac{m^{-4-(-4 / 3)} n^{1-2}}{\sqrt[3]{9}}=\frac{-12}{3}+\frac{4}{3} \\
& =\frac{m^{-8 / 3} n^{-1}}{\sqrt[3]{9}}=\frac{1}{\sqrt[3]{9} m^{2 / 3} n} \\
& \text { d) } 8 x^{3} y^{-1} \\
& =\frac{5 x^{\frac{9}{4}-3} y^{2-(-1)}}{4}=\frac{-3}{4} \\
& =\frac{5 x^{-3 / 4} y^{3}}{4} \\
& =\frac{5 y^{3}}{4 x^{3 / 4}} \\
& \text { f) } \frac{\left(5 x^{0} y^{4}\right)^{-4}}{1} \cdot y^{\frac{1}{2}} \\
& =\frac{1}{\left(5 x^{0} y^{4}\right)^{4}} \cdot y^{1 / 2} \\
& =\frac{1}{5^{4} x^{0.4} y^{4.4}} \cdot y^{\frac{1}{2}} \\
& \frac{1}{2}-\frac{16^{2}}{T_{92}} \\
& \begin{array}{l}
=\frac{y^{\frac{1}{2}-16}}{625}=\frac{1}{2}-\frac{32}{2} \\
=\frac{y^{-3 / 2 / 2}}{625}=\frac{1}{625 y^{3 / 2}}=-\frac{31}{2}
\end{array} \\
& \text { h) }\left(y^{-4} z^{4}\right)^{3} \cdot\left(z y^{2}\right)^{-2} \\
& =y^{-4 \cdot 3} z^{4 \cdot 3} \cdot \frac{1}{\left(z y^{2}\right)^{2}} \\
& =\frac{y^{-12} z^{12}}{z^{2} y^{4}} \\
& =y^{-12-4} z^{12-2} \\
& =y^{-16} z^{10}=\frac{z^{10}}{y^{16}} \\
& \text { j) }\left(m^{4} n^{2} \cdot-4 m^{-1} n\right)^{3} \\
& =\left(-4 m^{4+(-1)} n^{2+1}\right)^{3} \\
& =\left(-4 m^{3} n^{3}\right)^{3} \\
& =(-4)^{3} m^{3 \cdot 3} n^{3 \cdot 3} \\
& =-64 m^{9} n^{9}
\end{aligned}
$$

k) $\frac{\left(m^{2}\right)^{-1}}{3 n^{2} \cdot m^{4} n}$

$$
=\frac{m^{2 \cdot(-1)}}{3 n^{2+1} m^{4}}
$$

$$
=\frac{m^{-2}}{3 n^{3} m^{4}}
$$

$$
=\frac{m^{-2-4}}{3 n^{3}}
$$

$$
=\frac{m^{-6}}{3 n^{3}}=\frac{1}{3 m^{6} n^{3}}
$$

$$
\text { m) }\left(\frac{6 x^{-3} y^{2}}{4 x^{2} \cdot x^{4} y^{-4}}\right)^{2}
$$

$$
=\left(\frac{3 x^{-3-2-4} y^{2-(-4)}}{2}\right)^{2}
$$

$$
=\left(\frac{3 x^{-9} y^{6}}{2}\right)^{2}
$$

$$
=\frac{3^{2} x^{-9 \cdot 2} y^{6 \cdot 2}}{2^{2}}
$$

$$
=\frac{9 y^{12}}{4 x^{18}}
$$

$$
\text { o) } \frac{\left(4 a b^{6}\right)^{2}}{3 b^{-1} \cdot 6 b^{-4}} b^{6}=1
$$

$$
=\frac{(4 a)^{2}}{18 b^{-1+(-4)}}
$$

$$
=\frac{16 a^{2}}{18 b^{-5}}
$$

$$
=\frac{8 a^{2} b^{5}}{9}
$$

$=\left(32 e^{4-(-1)} f^{\prime}\right)^{\frac{1}{5}}$
$=\left(32 e^{5} f^{\prime}\right)^{1 / 5}$
$=32^{1 / 5} e^{5 \cdot v_{5}} f^{1 \cdot v_{5}}$
$=\sqrt[5]{32} e f^{1 / s}$
$=2 e f^{1 / 5}$

$$
\text { n) } \frac{2 y x^{-2} \cdot 3 x}{\left(x^{2} y^{-1}\right)^{3}}
$$

$$
=\frac{6 y x^{-2+1}}{x^{2 \cdot 3} y^{-1 \cdot 3}}
$$

$$
=\frac{6 y x^{-1}}{x^{6} y^{-3}}
$$

$=\frac{6 y x^{-1}}{x^{6} y^{-3}}$

$$
=6 y^{1-(-3)} x^{-1-6}
$$

$=6 y^{1-(-3)} x^{-1-6}$

$$
=6 y^{4} x^{-7}
$$

$$
=\frac{6 y^{4}}{x^{7}}
$$

$$
\text { p) } \frac{8 x^{3} y^{3} z^{-2} \cdot x^{-3}}{\left(-2 x^{4} y^{-2}\right)^{2}}
$$

p

$$
=\frac{8 x^{3+(-3)} y^{3} z^{-2}}{(-2)^{2} x^{4-2} y^{-2 \cdot 2}}
$$

$=\frac{8 x^{3+(-3)} y^{3} z^{-2}}{(-2)^{2} x^{4 \cdot 2} y^{-2 \cdot 2}}$

$$
=\frac{8 x^{0} y^{3} z^{-2}}{4 x^{8} y^{-4}}
$$

$$
=\frac{2 y^{3-(-4)}}{x^{8} z^{2}}
$$

$$
=\frac{2 y^{7}}{x^{8} z^{2}}
$$

