Chapter 4 Final Review

$3\sqrt[4]{28}$

index

radicand

coefficient

Whole Radical

coefficient = 1

$\sqrt{140}$, $\sqrt[3]{48}$

Mixed Radical

coefficient ≠ 1

$2\sqrt{7}$, $-5\sqrt[3]{4}$

Example: Express a whole radical as a mixed radical (simplify the radical)

a) $\sqrt{80}$

largest perfect square

$= \sqrt{16 \cdot 5}$

$= \sqrt{16} \cdot \sqrt{5}$

$= 4\sqrt{5}$

b) $\sqrt[3]{24}$

$= \sqrt[3]{8 \cdot 3}$

$= \sqrt[3]{8} \cdot \sqrt[3]{3}$

$= 2\sqrt[3]{3}$

$1^3 = 1$

$2^3 = 8$

$3^3 = 27$

$4^3 = 64$

$5^3 = 125$

Example: Express a mixed radical as a whole radical

a) $5\sqrt{6}$

exponent is the same as the index

$= \sqrt{5^2 \cdot 6}$

$= \sqrt{25 \cdot 6}$

$= \sqrt{150}$

b) $4\sqrt[3]{9}$

$= \sqrt[3]{4^3 \cdot 9}$

$= \sqrt[3]{64 \cdot 9}$

$= \sqrt[3]{576}$

Powers with Rational Exponents

exponent

$x^{\frac{1}{n}} = \sqrt[n]{x^1}$

index

$x^{\frac{m}{n}} = \sqrt[n]{x^m}$ or $\underbrace{\left(\sqrt[n]{x}\right)^m}_{\text{preferred}}$

Example: Write as a radical

$0.2 = \frac{2}{10} = \frac{1}{5}$

a) $35^{\frac{1}{2}}$

$\sqrt[2]{35^1}$

b) $90^{0.2} = 90^{\frac{1}{5}}$

$\sqrt[5]{90}$

c) $18^{\frac{6}{5}}$

$\left(\sqrt[5]{18}\right)^6$

or

$\sqrt[5]{18^6}$

d) $\left(\frac{4}{9}\right)^{\frac{2}{3}}$

$\left(\sqrt[3]{\frac{4}{9}}\right)^2$

or

$\sqrt[3]{\left(\frac{4}{9}\right)^2}$

Example: Write in exponential form

a) $\sqrt{21}$

$21^{\frac{1}{2}}$

b) $\sqrt[3]{44}$

$44^{\frac{1}{3}}$

c) $\left(\sqrt[5]{80}\right)^2$

$80^{\frac{2}{5}}$

d) $\sqrt[3]{\left(\frac{4}{9}\right)^5}$

$\left(\frac{4}{9}\right)^{\frac{5}{3}}$

Powers with Negative Exponents:

$\frac{x^{-n}}{1} = \frac{1}{x^n}$ $\qquad$ $\frac{1}{x^{-n}} = \frac{x^n}{1}$

Rational Numbers with Negative Exponents

$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$

Example: Write in radical form and evaluate.

a) $4^{-2} = \frac{1}{4^2} = \frac{1}{16}$

b) $8^{-\frac{2}{3}} = \frac{1}{8^{2/3}} = \frac{1}{\left(\sqrt[3]{8}\right)^2}$

$= \frac{1}{(2)^2}$

$= \frac{1}{4}$

c) $\left(\frac{4}{5}\right)^{-3} = \left(\frac{5}{4}\right)^3$

$= \frac{5^3}{4^3}$

$= \frac{125}{64}$

d) $\left(\frac{4}{9}\right)^{-\frac{5}{2}}$

$= \left(\frac{9}{4}\right)^{\frac{5}{2}}$ → rational exponent does <u>not</u> flip

$= \left(\sqrt{\frac{9}{4}}\right)^5$

$= \left(\frac{3}{2}\right)^5 = \frac{243}{32}$

Simplifying Expressions using Exponent Laws

Product Rule	$(x^a)(x^b) = x^{a+b}$		**Power of a Quotient Rule**	$\left(\frac{x}{y}\right)^a = \frac{x^a}{y^a}$
Quotient Rule	$\frac{x^a}{x^b} = x^{a-b}$		**Zero Exponents**	$x^0 = 1$
Power Rule	$(x^a)^b = x^{ab}$		**Negative Exponents**	$x^{-a} = \frac{1}{x^a}$
Power of a Product Rule	$(xy)^a = x^a y^a$			

Example: Simplify and express your answer with positive exponents.

a) $(2^1x^{-2})^5$

$= 2^5 x^{-10}$ ← only this term will move

$= \frac{32}{x^{10}}$

b) $\frac{x^{-3}y^5}{x^1y^4}$

$= x^{-4} y^1$

$= \frac{y}{x^4}$

c) $\left(\frac{-6c^3}{a^{\frac{1}{2}}}\right)^{-2}$ → rewrite with positive exponent

$\left(\frac{a^{\frac{1}{2}}}{-6c^3}\right)^2$

$= \frac{a^{\frac{1}{2}(\frac{2}{1})}}{(-6)^{1(2)} c^{3(2)}}$

$= \frac{a^1}{(-6)^2 c^6}$

$= \frac{a}{36c^6}$

d) $\frac{(2r^{\frac{3}{2}}s^2)(r^{\frac{1}{2}}s^{-1})}{6r^3s^{-4}}$ } simplify numerator first

$= \frac{2r^{\frac{3}{2}+\frac{1}{2}} s^{2+(-1)}}{6r^3s^{-4}}$

$= \frac{2r^2s^1}{6r^3s^{-4}}$

$= \frac{1r^{2-3}s^{1-(-4)}}{3}$

$= \frac{r^{-1}s^5}{3} = \frac{s^5}{3r}$