

## Ch. 8 Review

1. a)  $y = x^2 + 2x - 2$  ①  $\rightarrow y = (x^2 + 2x + \frac{1}{4} - \frac{1}{4}) - 2$

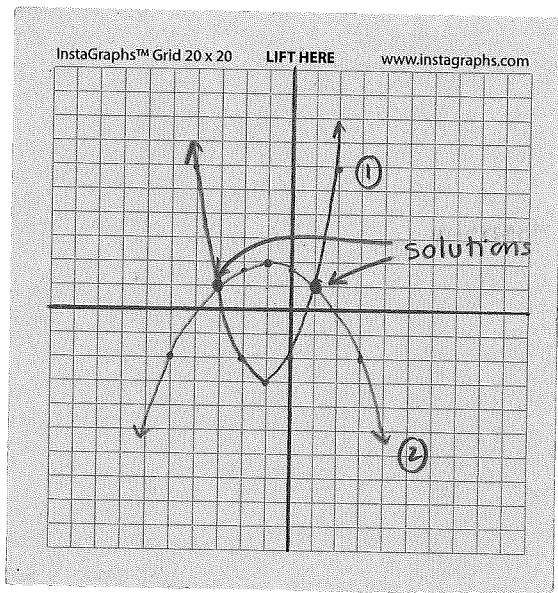
$y = -\frac{1}{4}(x+1) + 2$  ②

vertex  $(-1, 2)$

$y = (x^2 + 2x + 1) - 2 - 1$

$y = (x+1)(x+1) - 3$

$y = (x+1)^2 - 3 \rightarrow$  vertex  $(-1, 3)$



solutions:  $(-3, 1)$  and  $(1, 1)$

b)  $-x^2 - 8x - 2y - 8 = 0$  ①  $\rightarrow$

$3x + y = -4$  ②

$\downarrow$

$y = -3x - 4$  (straight line)

$\frac{2y}{2} = \frac{-x^2 - 8x - 8}{2}$

$y = -\frac{1}{2}x^2 - 4x - 4$

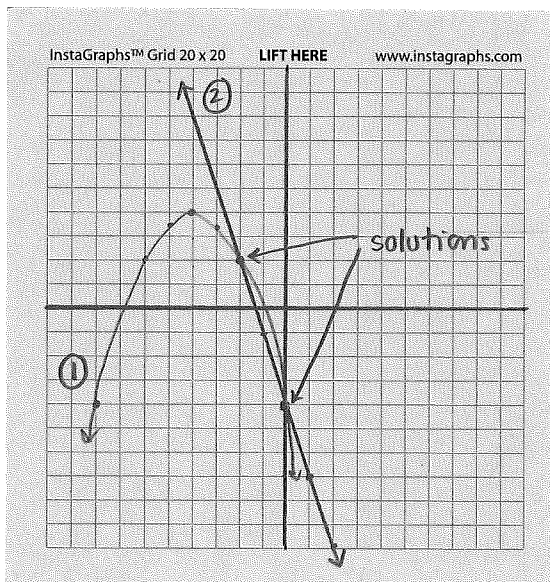
$= -\frac{1}{2}(x^2 + 8x + \frac{16}{2} - \frac{16}{2}) - 4$   
 $(\frac{8}{2})^2 = 16$

$= -\frac{1}{2}(x^2 + 8x + 16) - 4 + (-\frac{1}{2})(-16)$

$= -\frac{1}{2}(x+4)(x+4) + 4$

$y = -\frac{1}{2}(x+4)^2 + 4$

$\rightarrow$  vertex  $(-4, 4)$



solutions:  $(-2, 2)$  and  $(0, -4)$

2. a)  $3x + y = -9$  (1)  $\rightarrow y = -3x - 9$  (sub into (2))

$4x^2 - x + y = -9$  (2)

$4x^2 - x + (-3x - 9) = -9$

$4x^2 - 4x - 9 = -9$   
 $\quad\quad\quad +9 \quad +9$

$4x^2 - 4x = 0$

$4x(x-1) = 0$

$\downarrow \quad \downarrow$   
 $x=0 \quad x=1$

when  $x=0$ :  $y = -3(0) - 9$

$y = -9$

$(0, -9)$

$x=1$ :  $y = -3(1) - 9$

$y = -12$

$(1, -12)$

b)  $y - \frac{1}{2}x + 5 = 0$  (1)

$y = x^2 + 2x - 15 = 0$  (2) sub (2) into (1)

$2 \times [ (x^2 + 2x - 15) - \frac{1}{2}x + 5 = 0 ]$

to eliminate fractions

$2x^2 + 4x - 30 - x + 10 = 0$

$2x^2 + 3x - 20 = 0$

$2x^2 + 8x - 5x - 20 = 0$

$2x(x+4) - 5(x+4) = 0$

$(2x-5)(x+4) = 0$

$\downarrow \quad \downarrow$   
 $x = \frac{5}{2} \quad x = -4$

$(\frac{5}{2}, -\frac{15}{4})$  and  $(-4, -7)$

$\frac{8}{8} \times \frac{-5}{-5} = -40$   
 $\frac{8}{8} + \frac{-5}{-5} = 3$

when  $x = \frac{5}{2}$ :  $y = (\frac{5}{2})^2 + 2(\frac{5}{2}) - 15$

$= \frac{25}{4} - 10 \times \frac{4}{4}$

$= \frac{25-40}{4} = \frac{-15}{4}$

$x = -4$ :  $y = (-4)^2 + 2(-4) - 15$

$y = -7$

$$c) -x^2 - 7x - 2y + 2 = 0 \quad (1)$$

$$2x + y = -1 \quad (2) \rightarrow y = -2x - 1 \text{ sub into (1)}$$

$$-x^2 - 7x - 2(-2x - 1) + 2 = 0$$

$$-x^2 - 7x + 4x + 2 + 2 = 0$$

$$-x^2 - 3x + 4 = 0$$

$$-1(x^2 + 3x - 4) = 0$$

$$-1(x+4)(x-1) = 0$$

$$\downarrow \quad \downarrow$$

$$x = -4 \quad x = 1$$

$$\text{when } x = -4 : y = -2(-4) - 1 = 7$$

$$(-4, 7)$$

$$x = 1 : y = -2(1) - 1 = -3$$

$$(1, -3)$$

$$3. -12x^2 + 11y - 9x - 34 = 0 \quad (1)$$

$$a) y - 3x = 2 \quad (2)$$

$$-12x^2 + 11y - 9x - 34 = 0 \quad (1)$$

$$11 \cdot (y - 3x - 2 = 0) \quad (2)$$

$$-12x^2 + 11y - 9x - 34 = 0 \quad (1)$$

$$- (11y - 33x - 22 = 0) \quad (2)$$

$$-12x^2 + 24x - 12 = 0$$

$$-12(x^2 - 2x + 1) = 0$$

$$-12(x-1)(x-1) = 0$$

$$\downarrow \quad \downarrow$$

$$x = 1$$

$$\text{when } x = 1 : y - 3(1) = 2$$

$$y = 5$$

$$(1, 5)$$

$$b) \begin{cases} x + 2y = 46 & (1) \\ x^2 - 3y = 93 & (2) \end{cases}$$

$$2 \cdot (x^2 - 3y = 93) \quad (2)$$

$$3x + 6y = 138 \quad (1)$$

$$+ (2x^2 - 6y = 186) \quad (2)$$

$$2x^2 + 3x = 324$$

$$2x^2 + 3x - 324 = 0$$

$$(2)(-324) = -648$$

$$\frac{27}{27} \times \frac{-24}{-24} = -648$$

$$\frac{27}{27} + \frac{-24}{-24} = 3$$

$$2x^2 - 24x + 27x - 324 = 0$$

$$2x(x-12) + 27(x-12) = 0$$

$$(x-12)(2x+27) = 0$$

$$\downarrow$$

$$x = 12$$

$$\downarrow$$

$$x = -\frac{27}{2}$$

$$\text{when } x = 12 : 12 + 2y = 46$$

$$2y = 34 \quad y = 17$$

$$(12, 17)$$

$$x = -\frac{27}{2} : -\frac{27}{2} + 2y = 46$$

$$2y = \frac{119}{2} \quad y = \frac{119}{4}$$

$$\left(-\frac{27}{2}, \frac{119}{4}\right)$$

$$c) \quad -2x^2 + 3x - 2y + 1 = 0 \quad (1)$$

$$+ \left( \begin{array}{l} x + 2y - 3 = 0 \quad (2) \end{array} \right)$$

$$-2x^2 + 4x - 2 = 0$$

$$-2(x^2 - 2x + 1) = 0$$

$$-2(x-1)(x-1) = 0$$

$$\downarrow \quad \swarrow$$

$$x=1$$

when  $x=1$  :  $1 + 2y - 3 = 0$

$$2y = 2$$

$$y = 1$$

(1, 1)

$$4. a) \quad 2x^2 - 21x - y + 54 = 0 \quad (1)$$

$$x + y = 4 \quad (2)$$

$$2x^2 - 21x - y + 54 = 0 \quad (1)$$

$$+ \left( \begin{array}{l} x + y - 4 = 0 \quad (2) \end{array} \right)$$

$$2x^2 - 20x + 50 = 0$$

$$2(x^2 - 10x + 25) = 0$$

$$2(x-5)(x-5) = 0$$

$$\downarrow \quad \swarrow$$

$$x=5$$

when  $x=5$  :  $5 + y - 4 = 0$

$$y = -1$$

(5, -1)

$$c) \quad 3x^2 + 4x - y - 8 = 0 \quad (1)$$

$$y + 3 = 2x^2 + 4x \quad (2)$$

$$3x^2 + 4x - y - 8 = 0 \quad (1)$$

$$+ \left( \begin{array}{l} -2x^2 - 4x + y + 3 = 0 \quad (2) \end{array} \right)$$

$$x^2 - 5 = 0$$

$$x = \pm \sqrt{5}$$

$$(\pm 2.236)$$

$$b) \quad -x^2 + 8x + y - 9 = 0 \quad (1)$$

$$- \left( -x^2 + 32x + y - 33 = 0 \right) \quad (2)$$

$$-24x + 24 = 0$$

$$-24x = -24$$

$$x = 1$$

when  $x=1$  :  $-(1)^2 + 8(1) + y - 9 = 0$

$$-1 + 8 + y - 9 = 0$$

$$y = 2$$

(1, 2)

when  $x = -\sqrt{5} (-2.236)$

$$y = 2(-\sqrt{5})^2 + 4(-\sqrt{5}) - 3$$

$$= -1.944$$

$$x = +\sqrt{5}$$

$$y = 2(\sqrt{5})^2 + 4(\sqrt{5}) - 3$$

$$= 15.944$$

(-2.236, -1.944) and (2.236, 15.944)