Unit 3 – Kinematics

Kinematics is the study of how objects move.

A. Variables

In this unit, we will be dealing with a lot of different variables.

$\vec{v}_o$ or $\vec{v}_i$ initial velocity (m/s, km/hr)

$\vec{v}_f$ final velocity (m/s, km/hr)

$\vec{d}$ displacement (m)

$\vec{a}$ acceleration (m/s²)

t time (s)

convert to m/s first

$km/hr \xrightarrow[\div 3.6]{} m/s$

B. Sign Convention

Since we are working with vector quantities, we must define our sign convention.

↑ up/North positive

↓ down/South falling dropping negative

→ right/East positive

← left/West negative

C. Velocity and Acceleration

So far, we have seen 2 basic equations of motion:

Average Velocity:

$$\vec{V} = \frac{\Delta \vec{d}}{\Delta t}$$

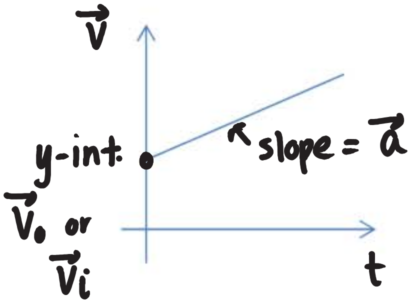
slope of $\vec{d}$ vs t graph

Average Acceleration:

$$\vec{a} = \frac{\Delta \vec{V}}{\Delta t}$$

slope of a $\vec{V}$ vs t graph

D. Velocity of an Object with Constant Acceleration



Remember from grade 10 that the equation of a line is $\overset{\vec{V}_f}{y} = \underset{\substack{\text{slope}\\(\vec{a})}}{m}\underset{t}{x}\overset{\text{y-int}}{\boxed{+b}}$

We can use the following equation to describe the velocity vs time graph:

$$\vec{V}_f = \vec{a}\cdot t + \vec{V}_o$$

or

$$\vec{V}_f = \vec{a}t + \vec{V}_i$$

$$\boxed{\vec{V}_f = \vec{V}_o + \vec{a}t}$$

EQUATION 1

Example 1: A motorcycle traveling $25\ m/s$ [E] applies its breaks. If it $1.2\ s$ takes to stop, determine the deceleration needed to stop.

$\vec{V}_o = {}^{+}25\ m/s$

$t = 1.2s$

$\vec{V}_f = 0$ (stops)

$\vec{a} = ?$

$$\vec{V}_f = \vec{V}_o + \vec{a}t$$

$$0 = 25 + \vec{a}(1.2)$$

$$-25 \qquad -25$$

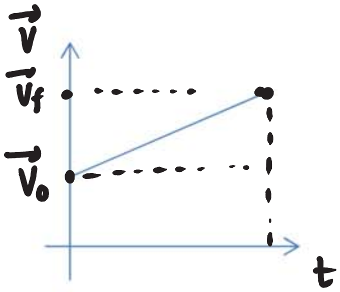
$$\frac{-25}{1.2} = \frac{1.2\vec{a}}{1.2}$$

$$\boxed{-20.8\ m/s^2 = \vec{a}}$$

means he's slowing down (opposing the direction of motion)

$-21\ m/s^2$

E. Displacement of an Object with Constant Acceleration



Recall from Unit 2, the area under the "curve" of a velocity vs time graph represents displacement.

$$\boxed{\vec{d} = \frac{1}{2}(\vec{V}_f + \vec{V}_o)t}$$

EQUATION 2

Example 2: What is the displacement of a train as it accelerates uniformly from $+11\ m/s$ to $+33\ m/s$ in 20 seconds?

$\vec{V}_o = +11\ m/s$ $\qquad \vec{d} = ?$

$\vec{V}_f = +33\ m/s$

$t = 20\ s$

$$\vec{d} = \frac{1}{2}(\vec{V}_f + \vec{V}_o)\,t$$

$$= \frac{1}{2}(33 + 11)(20)$$

$$= \frac{1}{2}(44)(20)$$

$$\boxed{\vec{d} = +440\ m}$$

Practice: p.69 #9 – 12, p.72 #13 – 16

E. Displacement cont.

Now, if we substitute the final velocity ($v_f = v_o + at$) from the first equation into the second equation ($d = \frac{1}{2}(v_f + v_o)t$); we end up with:

$$\boxed{\vec{d} = \vec{V}_o t + \frac{1}{2}\vec{a}t^2}$$

EQUATION 3

we don't need to know $\underline{V_f}$ to use this equation.

Example 3: What is the displacement of a car starting from rest if it accelerates at $+6.1\ m/s^2$ for $7.0\ s$.

$\vec{V}_o = 0$ (starting from rest)

$\vec{a} = +6.1\ m/s^2$

$t = 7.0\ s$

$\vec{d} = ?$

$$\vec{d} = \vec{V}_o t + \frac{1}{2}\vec{a}t^2$$

$$= (0)\cancel{(7.0)} + \frac{1}{2}(6.1)(7.0)^2$$

$$= +149.45\ m$$

$$\boxed{\vec{d} = 150\ m \quad \text{or} \quad 149\ m}$$

Now, if we combine the first and second equations again but this time the second one is solved for time; we end up with:

$$\boxed{\vec{V}_f^2 = \vec{V}_o^2 + 2\vec{a}\vec{d}}$$

EQUATION 4

we don't need $\underline{\text{time}}$ to use this equation.

Example 4: An airplane must reach a velocity of $71\ m/s$ for takeoff. If the runway is $1.0\ km$ long, what must its constant acceleration be?

$\vec{V}_f = 71\ m/s$ $\qquad \vec{a} = ?$

$\vec{V}_o = 0$ (assumed started from rest)

$\vec{d} = 1.0\ km = 1000\ m$

$$\vec{V}_f^2 = \vec{V}_o^2 + 2\vec{a}\vec{d}$$

$$(71)^2 = (0)^2 + 2\vec{a}(1000)$$

$$\frac{5041}{2000} = \frac{2000\vec{a}}{2000}$$

$$2.5205 = \vec{a}$$

$$\boxed{\vec{a} = 2.5\ m/s^2 \quad \text{or} \quad 2.52\ m/s^2}$$

Practice: p.74 #17-20, p.75 #21-23