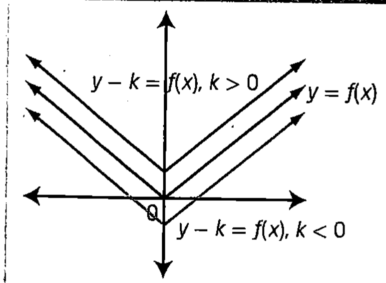
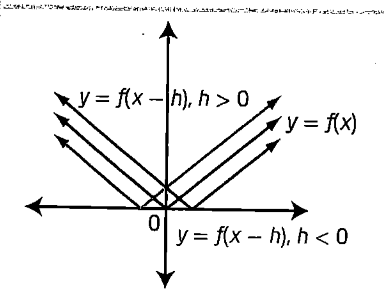
Key Ideas

- Translations are transformations that shift all points on the graph of a function up, down, left, and right without changing the shape or orientation of the graph.
- The table summarizes translations of the function $y = f(x)$.

Function	Transformation from $y = f(x)$	Mapping	Example
$y - k = f(x)$ or $y = f(x) + k$	A vertical translation If $k > 0$, the translation is up. If $k < 0$, the translation is down.	$(x, y) \rightarrow (x, y + k)$	
$y = f(x - h)$	A horizontal translation If $h > 0$, the translation is to the right. If $h < 0$, the translation is to the left.	$(x, y) \rightarrow (x + h, y)$	

- A sketch of the graph of $y - k = f(x - h)$, or $y = f(x - h) + k$, can be created by translating key points on the graph of the base function $y = f(x)$.

Check Your Understanding

Practise

1. For each function, state the values of h and k, the parameters that represent the horizontal and vertical translations applied to $y = f(x)$.

a) $y - 5 = f(x)$

b) $y = f(x) - 4$

c) $y = f(x + 1)$

d) $y + 3 = f(x - 7)$

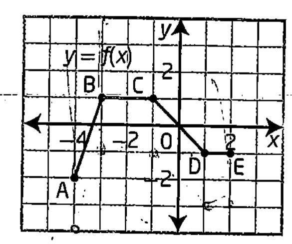
e) $y = f(x + 2) + 4$

2. Given the graph of $y = f(x)$ and each of the following transformations,

- state the coordinates of the image points A′, B′, C′, D′ and E′
- sketch the graph of the transformed function

a) $g(x) = f(x) + 3$ **b)** $h(x) = f(x - 2)$

c) $s(x) = f(x + 4)$ **d)** $t(x) = f(x) - 2$



3. Describe, using mapping notation, how the graphs of the following functions can be obtained from the graph of $y = f(x)$.

a) $y = f(x + 10)$

b) $y + 6 = f(x)$

c) $y = f(x - 7) + 4$

d) $y - 3 = f(x - 1)$

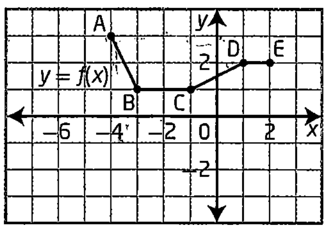
4. Given the graph of $y = f(x)$, sketch the graph of the transformed function. Describe the transformation that can be applied to the graph of $f(x)$ to obtain the graph of the transformed function. Then, write the transformation using mapping notation.

a) $r(x) = f(x + 4) - 3$

b) $s(x) = f(x - 2) - 4$

c) $t(x) = f(x - 2) + 5$

d) $v(x) = f(x + 3) + 2$



Apply

5. For each transformation, identify the values of h and k. Then, write the equation of the transformed function in the form $y - k = f(x - h)$.

a) $f(x) = \frac{1}{x}$, translated 5 units to the left and 4 units up

b) $f(x) = x^2$, translated 8 units to the right and 6 units up

c) $f(x) = |x|$, translated 10 units to the right and 8 units down

d) $y = f(x)$, translated 7 units to the left and 12 units down

6. What vertical translation is applied to $y = x^2$ if the transformed graph passes through the point $(4, 19)$?

7. What horizontal translation is applied to $y = x^2$ if the translation image graph passes through the point $(5, 16)$?

8. Copy and complete the table.

Translation	Transformed Function	Transformation of Points
vertical	$y = f(x) + 5$	$(x, y) \rightarrow (x, y + 5)$
	$y = f(x + 7)$	$(x, y) \rightarrow (x - 7, y)$
	$y = f(x - 3)$	
	$y = f(x) - 6$	
horizontal and vertical	$y + 9 = f(x + 4)$	
horizontal and vertical		$(x, y) \rightarrow (x + 4, y - 6)$
		$(x, y) \rightarrow (x - 2, y + 3)$
horizontal and vertical	$y = f(x - h) + k$	

9. The graph of the function $y = x^2$ is translated 4 units to the left and 5 units up to form the transformed function $y = g(x)$.

a) Determine the equation of the function $y = g(x)$.

b) What are the domain and range of the image function?

c) How could you use the description of the translation of the function $y = x^2$ to determine the domain and range of the image function?

10. The graph of $f(x) = |x|$ is transformed to the graph of $g(x) = f(x - 9) + 5$.

a) Determine the equation of the function $g(x)$.

b) Compare the graph of $g(x)$ to the graph of the base function $f(x)$.

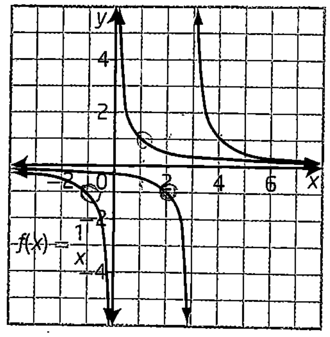
c) Determine three points on the graph of $f(x)$. Write the coordinates of the image points if you perform the horizontal translation first and then the vertical translation.

d) Using the same original points from part c), write the coordinates of the image points if you perform the vertical translation first and then the horizontal translation.

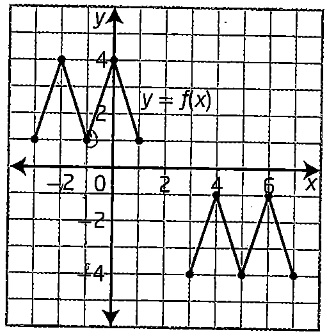
e) What do you notice about the coordinates of the image points from parts c) and d)? Is the order of the translations important?

11. The graph of the function drawn in red is a translation of the original function drawn in blue. Write the equation of the translated function in the form $y - k = f(x - h)$.

a)



b)

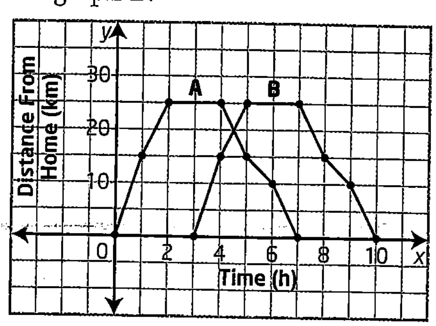


12. Janine is an avid cyclist. After cycling to a lake and back home, she graphs her distance versus time (graph A).

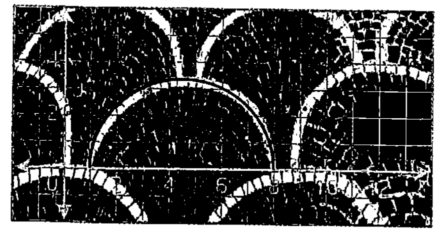
a) If she left her house at 12 noon, briefly describe a possible scenario for Janine's trip.

b) Describe the differences it would make to Janine's cycling trip if the graph of the function were translated, as shown in graph B.

c) The equation for graph A could be written as $y = f(x)$. Write the equation for graph B.



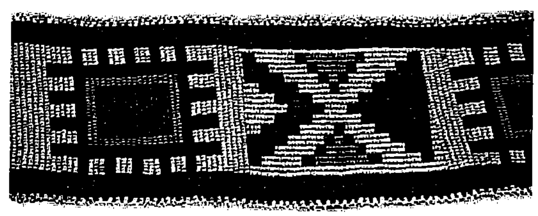
13. Architects and designers often use translations in their designs. The image shown is from an Italian roadway.



a) Use the coordinate plane overlay with the base semicircle shown to describe the approximate transformations of the semicircles.

b) If the semicircle at the bottom left of the image is defined by the function $y = f(x)$, state the approximate equations of three other semicircles.

14. This Pow Wow belt shows a frieze pattern where a particular image has been translated throughout the length of the belt.



a) With or without technology, create a design using a pattern that is a function. Use a minimum of four horizontal translations of your function to create your own frieze pattern.

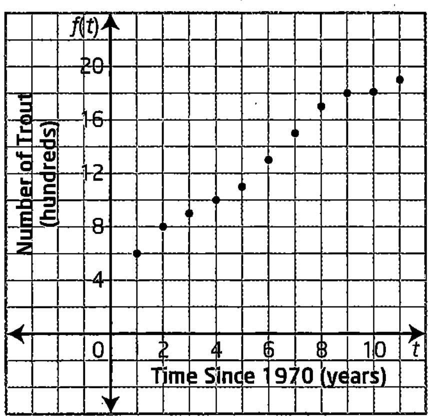
b) Describe the translation of your design in words and in an equation of the form $y = f(x - h)$.

Did You Know?

In First Nations communities today, Pow Wows have evolved into multi-tribal festivals. Traditional dances are performed by men, women, and children. The dancers wear traditional regalia specific to their dance style and nation of origin.

15. Michele Lake and Coral Lake, located near the Columbia Ice Fields, are the only two lakes in Alberta in which rare golden trout live.

Suppose the graph represents the number of golden trout in Michelle Lake in the years since 1970.



Let the function $f(t)$ represent the number of fish in Michelle Lake since 1970.

Describe an event or a situation for the fish population that would result in the following transformations of the graph. Then, use function notation to represent the transformation.

a) a vertical translation of 2 units up

b) a horizontal translation of 3 units to the right

16. Paul is an interior house painter. He determines that the function $n = f(A)$ gives the number of gallons, n, of paint needed to cover an area, A, in square metres. Interpret $n = f(A) + 10$ and $n = f(A + 10)$ in this context.

Extend

17. The graph of the function $y = x^2$ is translated to an image parabola with zeros 7 and 1.

a) Determine the equation of the image function.

b) Describe the translations on the graph of $y = x^2$.

c) Determine the y-intercept of the translated function.

18. Use translations to describe how the graph of $y = \frac{1}{x}$ compares to the graph of each function.

a) $y - 4 = \frac{1}{x}$

b) $y = \frac{1}{x + 2}$

c) $y - 3 = \frac{1}{x - 5}$

d) $y = \frac{1}{x + 3} - 4$

19. a) Predict the relationship between the graph of $y = x^3 - x^2$ and the graph of $y + 3 = (x - 2)^3 - (x - 2)^2$.

b) Graph each function to verify your prediction.

Create Connections

C1 The graph of the function $y = f(x)$ is transformed to the graph of $y = f(x - h) + k$.

a) Show that the order in which you apply translations does not matter. Explain why this is true.

b) How are the domain and range affected by the parameters h and k?

C2 Complete the square and explain how to transform the graph of $y = x^2$ to the graph of each function.

a) $f(x) = x^2 + 2x + 1$

b) $g(x) = x^2 - 4x + 3$

C3 The roots of the quadratic equation $x^2 - x - 12 = 0$ are -3 and 4. Determine the roots of the equation $(x - 5)^2 - (x - 5) - 12 = 0$.

C4 The function $f(x) = x + 4$ could be a vertical translation of 4 units up or a horizontal translation of 4 units to the left. Explain why.

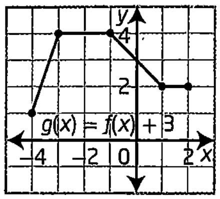
Answers

Chapter 1 Function Transformations

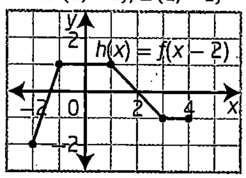
1.1 Horizontal and Vertical Translations, pages 12 to 15

1. a) $h = 0, k = 5$ **b)** $h = 0, k = -4$ **c)** $h = -1, k = 0$
d) $h = 7, k = -3$ **e)** $h = -2, k = 4$

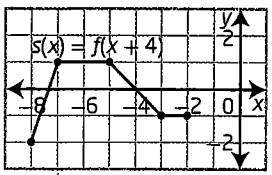
2. a) A′(−4, 1), B′(−3, 4), C′(−1, 4), D′(1, 2), E′(2, 2)



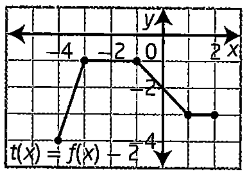
b) A′(−2, −2), B′(−1, 1), C′(1, 1), D′(3, −1), E′(4, −1)



c) A′(−8, −2), B′(−7, 1), C′(−5, 1), D′(−3, −1), E′(−2, −1)

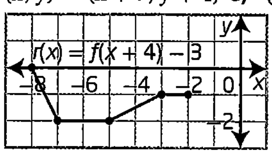


d) A′(−4, −4), B′(−3, −1), C′(−1, −1), D′(1, −3), E′(2, −3)



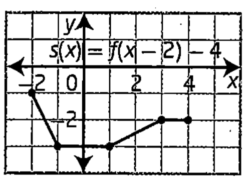
3. a) $(x, y) \rightarrow (x - 10, y)$ **b)** $(x, y) \rightarrow (x, y - 6)$
c) $(x, y) \rightarrow (x + 7, y + 4)$ **d)** $(x, y) \rightarrow (x + 1, y + 3)$

4. a)



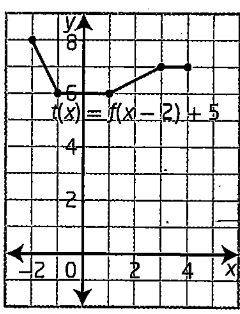
a vertical translation of 3 units down and a horizontal translation of 4 units left; $(x, y) \rightarrow (x - 4, y - 3)$

b)



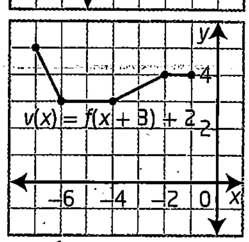
a vertical translation of 4 units down and a horizontal translation of 2 units right; $(x, y) \rightarrow (x + 2, y - 4)$

c)



a vertical translation of 5 units up and a horizontal translation of 2 units right; $(x, y) \rightarrow (x + 2, y + 5)$

d)



a vertical translation of 2 units up and a horizontal translation of 3 units left; $(x, y) \rightarrow (x - 3, y + 2)$

5. a) $h = -5, k = 4; y - 4 = f(x + 5)$
b) $h = 8, k = 6; y - 6 = f(x - 8)$
c) $h = 10, k = -8; y + 8 = f(x - 10)$
d) $h = -7, k = -12; y + 12 = f(x + 7)$

6. It has been translated 3 units up.

7. It has been translated 1 unit right.

8.

Translation	Transformed Function	Transformation of Points
vertical	$y = f(x) + 5$	$(x, y) \rightarrow (x, y + 5)$
horizontal	$y = f(x + 7)$	$(x, y) \rightarrow (x - 7, y)$
horizontal	$y = f(x - 3)$	$(x, y) \rightarrow (x + 3, y)$
vertical	$y = f(x) - 6$	$(x, y) \rightarrow (x, y - 6)$
horizontal and vertical	$y + 9 = f(x + 4)$	$(x, y) \rightarrow (x - 4, y - 9)$
horizontal and vertical	$y = f(x - 4) - 6$	$(x, y) \rightarrow (x + 4, y - 6)$
horizontal and vertical	$y = f(x + 2) + 3$	$(x, y) \rightarrow (x - 2, y + 3)$
horizontal and vertical	$y = f(x - h) + k$	$(x, y) \rightarrow (x + h, y + k)$

9. a) $y = (x + 4)^2 + 5$ **b)** $\{x \mid x \in \mathrm{R}\}, \{y \mid y \geq 5, y \in \mathrm{R}\}$
c) To determine the image function's domain and range, add the horizontal and vertical translations to the domain and range of the base function. Since the domain is the set of real numbers, nothing changes, but the range does change.

10. a) $g(x) = |x - 9| + 5$
b) The new graph is a vertical and horizontal translation of the original by 5 units up and 9 units right.
c) Example: (0, 0), (1, 1), (2, 2) → (9, 5), (10, 6), (11, 7)
d) Example: (0, 0), (1, 1), (2, 2) → (9, 5), (10, 6), (11, 7)
e) The coordinates of the image points from parts c) and d) are the same. The order that the translations are made does not matter.

11. a) $y = f(x - 3)$ **b)** $y + 5 = f(x - 6)$

12. a) Example: It takes her 2 h to cycle to the lake, 25 km away. She rests at the lake for 2 h and then returns home in 3 h.
b) This translation shows what would happen if she left the house at a later time.
c) $y = f(x - 3)$

13. a) Example: Translated 8 units right.
b) Example: $y = f(x - 8)$, $y = f(x - 4) + 3.5$, $y = f(x + 4) + 3.5$

14. a) Example: A repeating X by using two linear equations $y = \pm x$.
b) Example: $y = f(x - 3)$. The translation is horizontal by 3 units right.

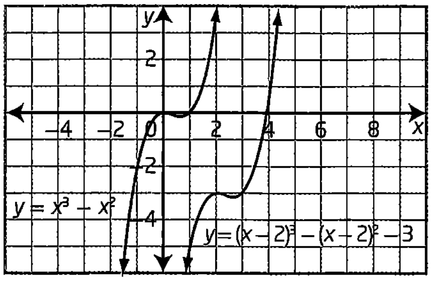
15. a) The transformed function starts with a higher number of trout in 1970. $y = f(t) + 2$
b) The transformed function starts in 1974 instead of 1971. $y = f(t - 3)$

16. The first case, $n = f(A) + 10$, represents the number of gallons he needs for a given area plus 10 more gallons. The second case, $n = f(A + 10)$, represents how many gallons he needs to cover an area A less 10 units of area.

17. a) $y = (x - 7)(x - 1)$ or $y = (x - 4)^2 - 9$
b) Horizontal translation of 4 units right and vertical translation of 9 units down.
c) y-intercept 7

18. a) The original function is 4 units lower.
b) The original function is 2 units to the right.
c) The original function is 3 units lower and 5 units left.
d) The original function is 4 units higher and 3 units right.

19. a) The new graph will be translated 2 units right and 3 units down.
b)



C1 a) $y = f(x) \rightarrow y = f(x - h) \rightarrow y = f(x - h) + k$. Looking at the problem in small steps, it is easy to see that it does not matter which way the translations are done since they do not affect the other translation.
b) The domain is shifted by h and the range is shifted by k.

C2 a) $f(x) = (x + 1)^2$; horizontal translation of 1 unit left
b) $g(x) = (x - 2)^2 - 1$; horizontal translation of 2 units right and 1 unit down

C3 The roots are 2 and 9.

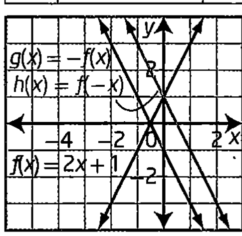
C4 The 4 can be taken as h or k in this problem. If it is h then it is -4, which makes it in the left direction.

1.2 Reflections and Stretches, pages 28 to 31

1. a)

x	$f(x) = 2x + 1$	$g(x) = -f(x)$	$h(x) = f(-x)$
−4	−7	7	9
−2	−3	3	5
0	1	−1	1
2	5	−5	−3
4	9	−9	−7

b)



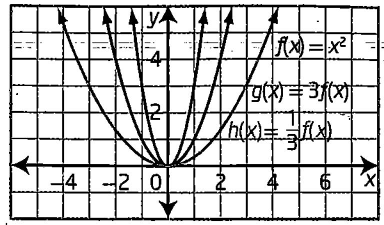
c) The y-coordinates of $g(x)$ have changed sign. The invariant point is $(-0.5, 0)$. The x-coordinates of $h(x)$ have changed sign. The invariant point is $(0, 1)$.

d) The graph of $g(x)$ is the reflection of the graph of $f(x)$ in the x-axis, while the graph of $h(x)$ is the reflection of the graph of $f(x)$ in the y-axis.

2. a)

x	$f(x) = x^2$	$g(x) = 3f(x)$	$h(x) = \frac{1}{3}f(x)$
−6	36	108	12
−3	9	27	3
0	0	0	0
3	9	27	3
6	36	108	12

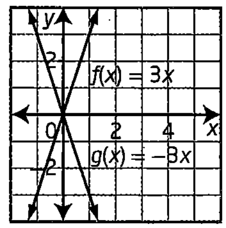
b)



c) The y-coordinates of $g(x)$ are three times larger. The invariant point is (0, 0). The y-coordinates of $h(x)$ are three times smaller. The invariant point is (0, 0).

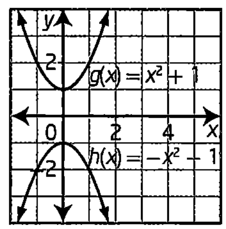
d) The graph of $g(x)$ is a vertical stretch by a factor of 3 of the graph of $f(x)$, while the graph of $h(x)$ is a vertical stretch by a factor of $\frac{1}{3}$ of the graph of $f(x)$.

3. a)



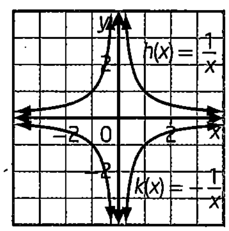
$g(x) = -3x$
$f(x)$: domain $\{x \mid x \in R\}$, range $\{y \mid y \in R\}$
$g(x)$: domain $\{x \mid x \in R\}$, range $\{y \mid y \in R\}$

b)



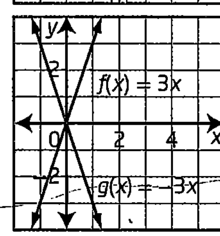
$h(x) = -x^2 - 1$
$g(x)$: domain $\{x \mid x \in R\}$, range $\{y \mid y \geq 1, y \in R\}$
$h(x)$: domain $\{x \mid x \in R\}$, range $\{y \mid y \leq -1, y \in R\}$

c)



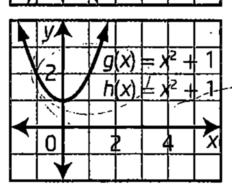
$k(x) = -\frac{1}{x}$
$h(x)$: domain $\{x \mid x \neq 0, x \in R\}$, range $\{y \mid y \neq 0, y \in R\}$
$k(x)$: domain $\{x \mid x \neq 0, x \in R\}$, range $\{y \mid y \neq 0, y \in R\}$

4. a)



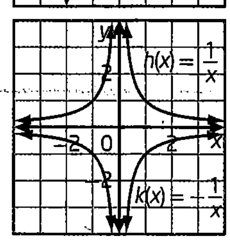
$g(x) = -3x$
$f(x)$: domain $\{x \mid x \in R\}$, range $\{y \mid y \in R\}$
$g(x)$: domain $\{x \mid x \in R\}$, range $\{y \mid y \in R\}$

b)



$h(x) = x^2 + 1$
$g(x)$: domain $\{x \mid x \in R\}$, range $\{y \mid y \geq 1, y \in R\}$
$h(x)$: domain $\{x \mid x \in R\}$, range $\{y \mid y \geq 1, y \in R\}$

c)



$k(x) = -\frac{1}{x}$
$h(x)$: domain $\{x \mid x \neq 0, x \in R\}$, range $\{y \mid y \neq 0, y \in R\}$
$k(x)$: domain $\{x \mid x \neq 0, x \in R\}$, range $\{y \mid y \neq 0, y \in R\}$