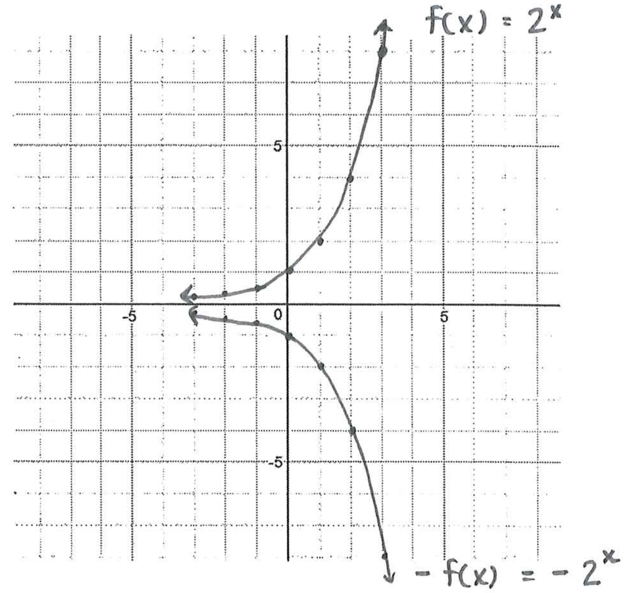
1.2 Reflections and Stretches

Example 1: Complete the table of values and graph the two functions on the same set of axes.

$-1 \cdot 2^x$

x	$f(x) = 2^x$	$-f(x) = -2^x$
-3	$\frac{1}{8}$	$-\frac{1}{8}$
-2	$\frac{1}{4}$	$-\frac{1}{4}$
-1	$\frac{1}{2}$	$-\frac{1}{2}$
0	1	-1
1	2	-2
2	4	-4
3	8	-8

$y = 2^{-3} = \frac{1}{2^3} = \frac{1}{8}$



Describe how the two graphs are related: - f(x) is a reflection over the x-axis of f(x).

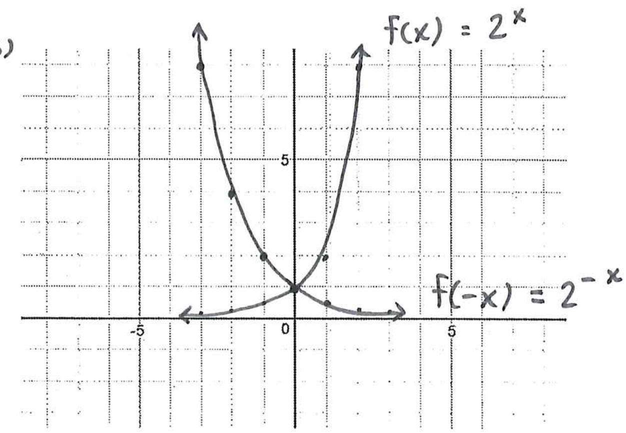
In general, the graph of $y = -f(x)$ is a *reflection* of the graph of $y = -f(x)$ in the x - axis.

Mapping Notation: $(x, y) \rightarrow (x, -y)$

Example 2: Complete the table of values and graph the two functions on the same set of axes.

x	$f(x) = 2^x$	$f(-x) = 2^{-x}$
-3	$\frac{1}{8}$	8
-2	$\frac{1}{4}$	4
-1	$\frac{1}{2}$	2
0	1	1
1	2	$\frac{1}{2}$
2	4	$\frac{1}{4}$
3	8	$\frac{1}{8}$

$2^{-(-3)} = 2^3$



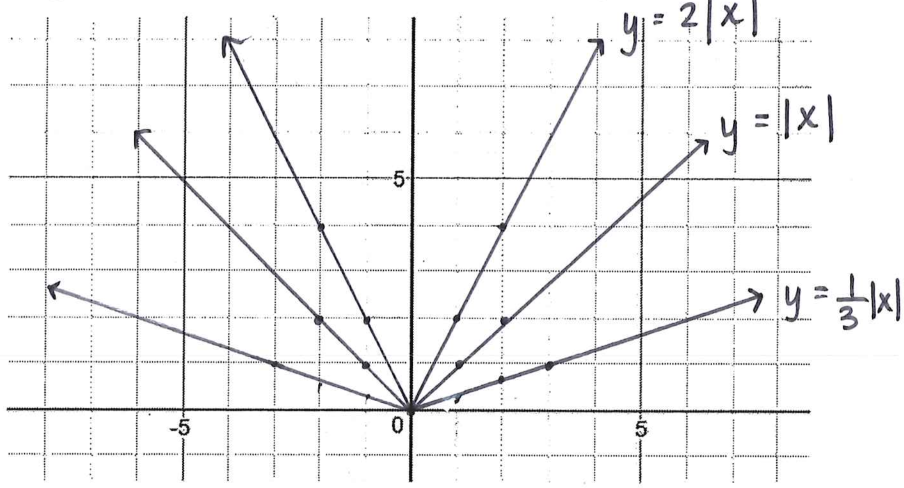
Describe how the two graphs are related: f(-x) is a reflection over the y-axis of f(x)

In general, the graph of $y = f(-x)$ is a *reflection* of the graph of $y = f(x)$ in the y - axis.

Mapping Notation: $(x, y) \rightarrow (-x, y)$

Example 3: Sketch the graph of $y=|x|$, $y=2|x|$, and $y=\frac{1}{3}|x|$ on the axes below.

x	$y=\|x\|$	$y=2\|x\|$	$y=\frac{1}{3}\|x\|$
-2	2	4	$\frac{2}{3}$
-1	1	2	$\frac{1}{3}$
0	0	0	0
1	1	2	$\frac{1}{3}$
2	2	4	$\frac{2}{3}$



In general, for any function $y=f(x)$, the graph of $y=af(x)$, where $\boldsymbol{a}$ is any real number results in a vertical stretch by a factor of "a"

Mapping Notation: $(x,y) \rightarrow (x, ay)$

Example 4: Graph the following functions on the axes below.

$y=\sqrt{x}$

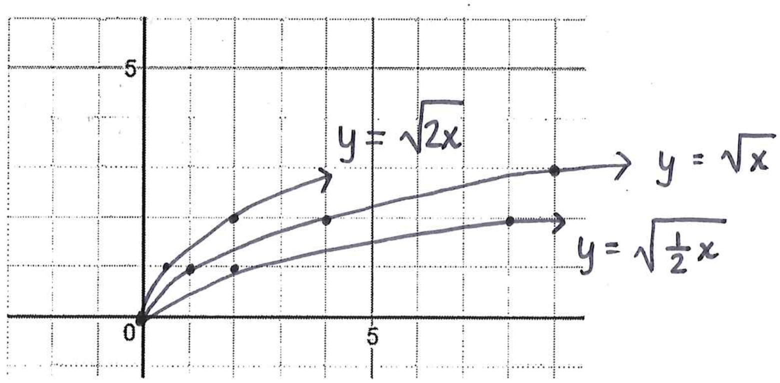
x	y
0	0
1	1
4	2
9	3

$y=\sqrt{2x}$

x	y
0	0
$\frac{1}{2}$	1
2	2

$y=\sqrt{\frac{1}{2}x}$

x	y
0	0
2	1
8	2



In general, for any function $y=f(x)$, the graph of $y=f(bx)$, where $\boldsymbol{b}$ is any real number results in a horizontal stretch by a factor of "$\frac{1}{b}$"

Mapping Notation: $(x,y) \rightarrow (\frac{x}{b}, y)$

Example 5: Given the graph of $y = f(x)$,

a) Sketch the graph of $g(x) = f(3x)$
b) Describe the transformation
c) State any invariant points (points that do not change even when a transformation occurs)
d) State the domain and range of the new function

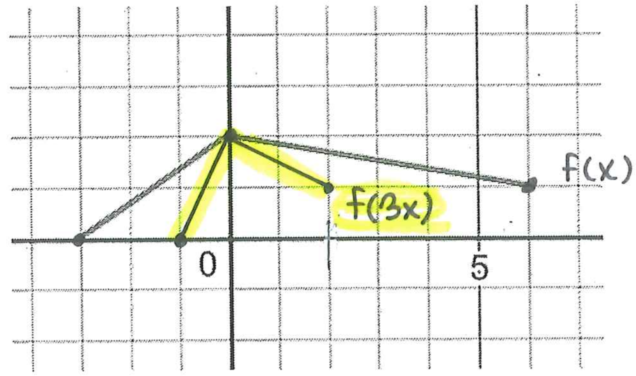
a) Graphing

$\cdot\frac{1}{3}$	x	y
-1	$\cancel{-3}$	0
0	$\cancel{0}$	2
2	$\cancel{6}$	1

↑ new values for f(3x)

b) transformation

horizontal stretch by a factor of $\frac{1}{3}$



c) invarient point (0,2)

d) Domain : $\{x \mid -1 \le x \le 2, x \in \mathbb{R}\}$

Range : $\{y \mid 0 \le y \le 2, y \in \mathbb{R}\}$

Example 6: Given the graph of $y = f(x)$,

a) Sketch the graph of $g(x) = f(-x)$
b) Describe the transformation
c) State any invariant points
d) State the domain and range of the new function

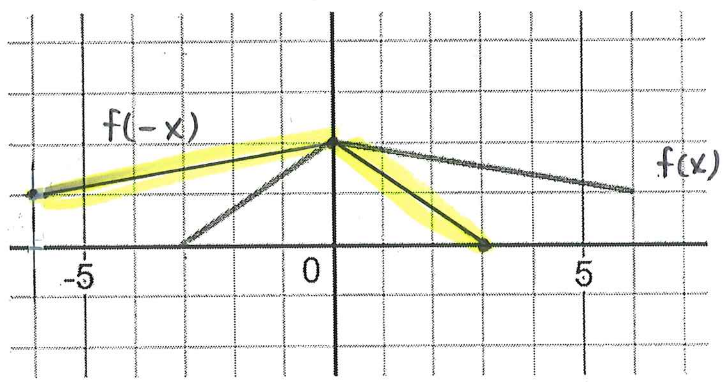
a) Graphing

$-1\cdot$	x	y
3	$\cancel{-3}$	0
0	$\cancel{0}$	2
-6	$\cancel{6}$	1

↑ new values for f(-x)

b) Transformation

Reflection in the y-axis



c) Invarient point

(0,2)

d) Domain : $\{x \mid -6 \le x \le 3, x \in \mathbb{R}\}$

Range : $\{y \mid 0 \le y \le 2, y \in \mathbb{R}\}$

Example 7: Given $f(x) = (x - 2)(x + 3)$, use transformations to determine the zeroes of each of the following functions.

zeroes/roots → values of x when $f(x) = 0$ roots

$$0 = (x-2)(x+3)$$

$x = 2$ $x = -3$

$(2,0)$ and $(-3,0)$

a) $g(x) = f(2x)$

horizontal stretch by a factor of $\frac{1}{2}$

(mult. x values by $\frac{1}{2}$)

$(x,y) \longrightarrow (\frac{1}{2}x, y)$

$(2,0) \longrightarrow (1,0)$

$(-3,0) \longrightarrow (-\frac{3}{2}, 0)$

b) $g(x) = f(-x)$

reflection in the y-axis

(mult. x-values by −1)

$(x,y) \longrightarrow (-x, y)$

$(2,0) \longrightarrow (-2,0)$

$(-3,0) \longrightarrow (3,0)$

c) $g(x) = 3f(x)$

vertical stretch by a factor of 3

(mult. y values by 3)

$(x,y) \longrightarrow (x, 3y)$

$(2,0) \longrightarrow (2,0)$

$(-3,0) \longrightarrow (-3,0)$

No change since values were zero.

d) $g(x) = -f(x)$

reflection in the x-axis

(mult. y-values by −1)

$(x,y) \longrightarrow (x, -y)$

$(2,0) \longrightarrow (2,0)$

$(-3,0) \longrightarrow (-3,0)$

No change

Practice: p.28 #1, 3 – 5, 7, 9ef, 14, 15