Check Your Understanding

Practise

1. a) Copy and complete the table of values for the given functions.

x	$f(x) = 2x + 1$	$g(x) = -f(x)$	$h(x) = f(-x)$
−4			
−2			
0			
2			
4			

b) Sketch the graphs of $f(x)$, $g(x)$, and $h(x)$ on the same set of coordinate axes.

c) Explain how the points on the graphs of $g(x)$ and $h(x)$ relate to the transformation of the function $f(x) = 2x + 1$. List any invariant points.

d) How is each function related to the graph of $f(x) = 2x + 1$?

2. a) Copy and complete the table of values for the given functions.

x	$f(x) = x^2$	$g(x) = 3f(x)$	$h(x) = \frac{1}{3}f(x)$
−6			
−3			
0			
3			
6			

b) Sketch the graphs of $f(x)$, $g(x)$, and $h(x)$ on the same set of coordinate axes.

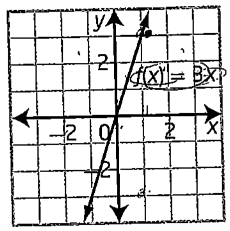
c) Explain how the points on the graphs of $g(x)$ and $h(x)$ relate to the transformation of the function $f(x) = x^2$. List any invariant points.

d) How is each function related to the graph of $f(x) = x^2$?

3. Consider each graph of a function.

- Copy the graph of the function and sketch its reflection in the x-axis on the same set of axes.
- State the equation of the reflected function in simplified form.
- State the domain and range of each function.

a)



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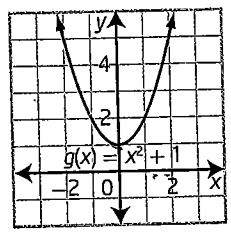

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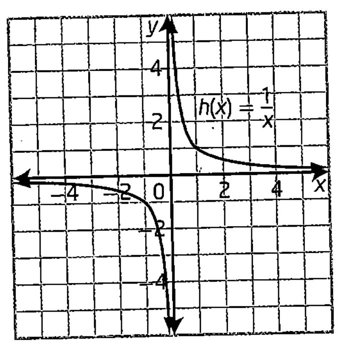


4. Consider each function in #3.

- Copy the graph of the function and sketch its reflection in the y-axis on the same set of axes.
- State the equation of the reflected function.
- State the domain and range for each function.

5. Use words and mapping notation to describe how the graph of each function can be found from the graph of the function $y = f(x)$.

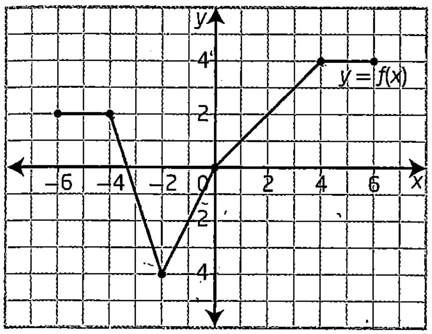
a) $y = 4f(x)$

b) $y = f(3x)$

c) $y = -f(x)$

d) $y = f(-x)$

6. The graph of the function $y = f(x)$ is vertically stretched about the x-axis by a factor of 2.

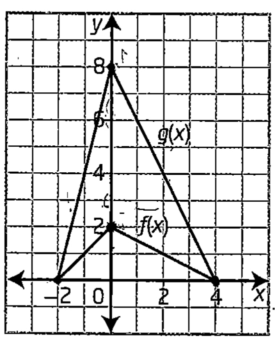


a) Determine the domain and range of the transformed function.

b) Explain the effect that a vertical stretch has on the domain and range of a function.

7. Describe the transformation that must be applied to the graph of $f(x)$ to obtain the graph of $g(x)$. Then, determine the equation of $g(x)$ in the form $y = af(bx)$.

a)



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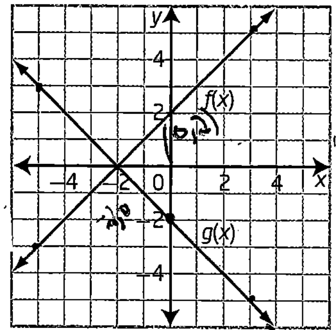
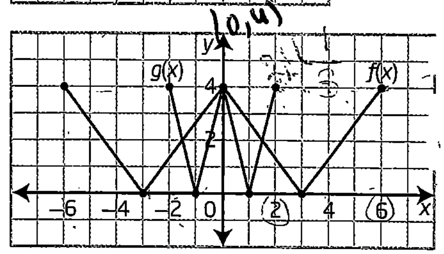
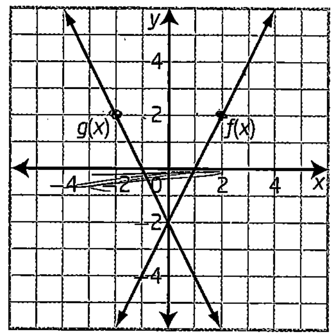


c)

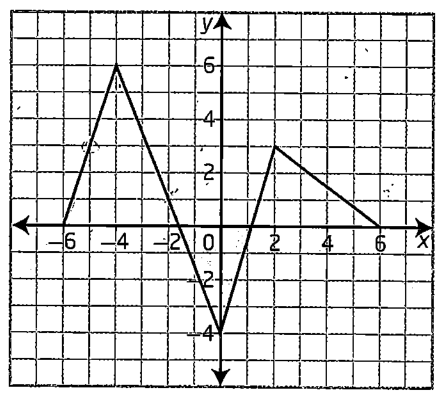


d)



Apply

8. A weaver sets up a pattern on a computer using the graph shown. A new line of merchandise calls for the design to be altered to $y = f(0.5x)$. Sketch the graph of the new design.



• translate
translate up 7

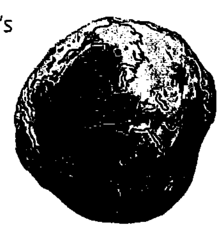
9. Describe what happens to the graph of a function $y = f(x)$ after the following changes are made to its equation.
   a) Replace x with $4x$.
   b) Replace x with $\frac{1}{4}x$.
   c) Replace y with $2y$.
   d) Replace y with $\frac{1}{4}y$.
   e) Replace x with $-3x$.
   f) Replace y with $-\frac{1}{3}y$.

10. Thomas and Sharyn discuss the order of the transformations of the graph of $y = -3|x|$ compared to the graph of $y = |x|$. Thomas states that the reflection must be applied first. Sharyn claims that the vertical stretch should be applied first.
   a) Sketch the graph of $y = -3|x|$ by applying the reflection first.
   b) Sketch the graph of $y = -3|x|$ by applying the stretch first.
   c) Explain your conclusions. Who is correct?

11. An object falling in a vacuum is affected only by the gravitational force. An equation that can model a free-falling object on Earth is $d = -4.9t^2$, where d is the distance travelled, in metres, and t is the time, in seconds. An object free falling on the moon can be modelled by the equation $d = -1.6t^2$.
   a) Sketch the graph of each function.
   b) Compare each function equation to the base function $d = t^2$.

Did You Know?

The actual strength of Earth's gravity varies depending on location.



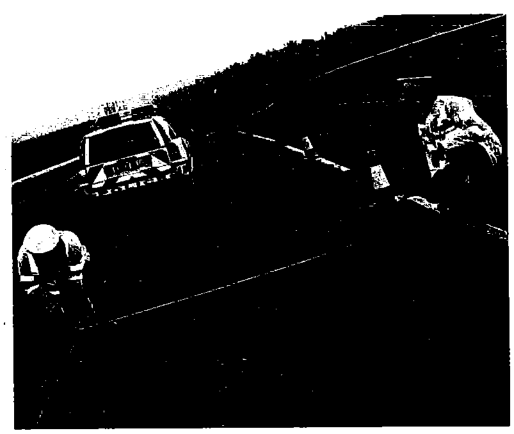
On March 17, 2009, the European Space Agency launched a gravity-mapping satellite called Gravity and Ocean Circulation Explorer (GOCE). The data transmitted from GOCE are being used to build a model of Earth's shape and a gravity map of the planet.

12. Explain the differences that occur in transforming the graph of the function $y = f(x)$ to the graph of the function $y = f(bx)$ as compared to transforming $y = f(x)$ to $y = af(x)$.

13. The speed of a vehicle the moment the brakes are applied can be determined by its skid marks. The length, D, in feet, of the skid mark is related to the speed, S, in miles per hour, of the vehicle before braking by the function $D = \frac{1}{30fn}S^2$, where f is the drag factor of the road surface and n is the braking efficiency as a decimal. Suppose the braking efficiency is 100% or 1.
   a) Sketch the graph of the length of the skid mark as a function of speed for a drag factor of 1, or $D = \frac{1}{30}S^2$.
   b) The drag factor for asphalt is 0.9, for gravel is 0.8, for snow is 0.55, and for ice is 0.25. Compare the graphs of the functions for these drag factors to the graph in part a).

Did You Know?

A technical accident investigator or reconstructionist is a specially trained police officer who investigates serious traffic accidents. These officers use photography, measurements of skid patterns, and other information to determine the cause of the collision and if any charges should be laid.



Extend

14. Consider the function $f(x) = (x + 4)(x - 3)$. Without graphing, determine the zeros of the function after each transformation.

a) $y = 4f(x)$

b) $y = f(-x)$

c) $y = f\left(\frac{1}{2}x\right)$

d) $y = f(2x)$

15. The graph of a function $y = f(x)$ is contained completely in the fourth quadrant. Copy and complete each statement.

a) If $y = f(x)$ is transformed to $y = -f(x)$, it will be in quadrant ■.

b) If $y = f(x)$ is transformed to $y = f(-x)$, it will be in quadrant ■.

c) If $y = f(x)$ is transformed to $y = 4f(x)$, it will be in quadrant ■.

d) If $y = f(x)$ is transformed to $y = f\left(\frac{1}{4}x\right)$, it will be in quadrant ■.

16. Sketch the graph of $f(x) = |x|$ reflected in each line.

a) $x = 3$

b) $y = -2$

Create Connections

C1 Explain why the graph of $g(x) = f(bx)$ is a horizontal stretch about the y-axis by a factor of $\frac{1}{b}$, for $b > 0$, rather than a factor of b.

C2 Describe a transformation that results in each situation. Is there more than one possibility?

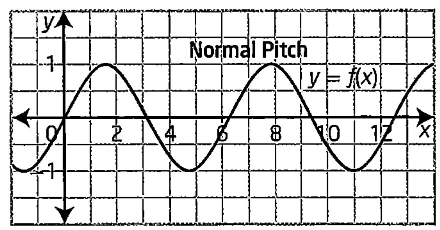
a) The x-intercepts are invariant points.

b) The y-intercepts are invariant points.

C3 A point on the function $f(x)$ is mapped onto the image point on the function $g(x)$. Copy and complete the table by describing a possible transformation of $f(x)$ to obtain $g(x)$ for each mapping.

$f(x)$	$g(x)$	Transformation
(5, 6)	(5, −6)	
(4, 8)	(−4, 8)	
(2, 3)	(2, 12)	
(4, −12)	(2, −6)	

C4 Sound is a form of energy produced and transmitted by vibrating matter that travels in waves. Pitch is the measure of how high or how low a sound is. The graph of $f(x)$ demonstrates a normal pitch. Copy the graph, then sketch the graphs of $y = f(3x)$, indicating a higher pitch, and $y = f\left(\frac{1}{2}x\right)$, for a lower pitch.



Did You Know?

The *pitch* of a sound wave is directly related to its *frequency*. A high-pitched sound has a high frequency (a mosquito). A low-pitched sound has a low frequency (a fog-horn).

A healthy human ear can hear frequencies in the range of 20 Hz to 20 000 Hz.

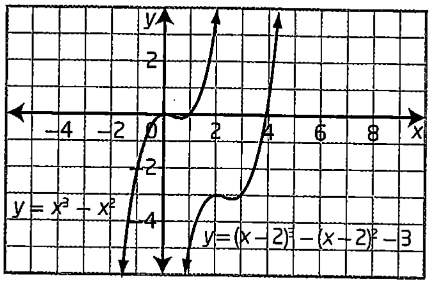
C5 a) Write the equation for the general term of the sequence −10, −6, −2, 2, 6,....

b) Write the equation for the general term of the sequence 10, 6, 2, −2, −6,....

c) How are the graphs of the two sequences related?

18. a) The original function is 4 units lower.
b) The original function is 2 units to the right.
c) The original function is 3 units lower and 5 units left.
d) The original function is 4 units higher and 3 units right.

19. a) The new graph will be translated 2 units right and 3 units down.
b)



C1 a) $y = f(x) \rightarrow y = f(x - h) \rightarrow y = f(x - h) + k$. Looking at the problem in small steps, it is easy to see that it does not matter which way the translations are done since they do not affect the other translation.
b) The domain is shifted by h and the range is shifted by k.

C2 a) $f(x) = (x + 1)^2$; horizontal translation of 1 unit left
b) $g(x) = (x - 2)^2 - 1$; horizontal translation of 2 units right and 1 unit down

C3 The roots are 2 and 9.

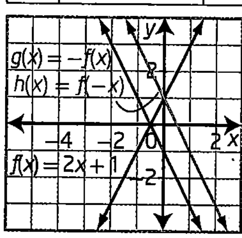
C4 The 4 can be taken as h or k in this problem. If it is h then it is −4, which makes it in the left direction.

1.2 Reflections and Stretches, pages 28 to 31

1. a)

x	$f(x) = 2x + 1$	$g(x) = -f(x)$	$h(x) = f(-x)$
−4	−7	7	9
−2	−3	3	5
0	1	−1	1
2	5	−5	−3
4	9	−9	−7

b)



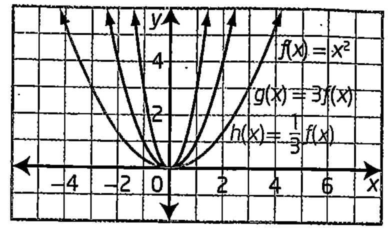
c) The y-coordinates of $g(x)$ have changed sign. The invariant point is (−0.5, 0). The x-coordinates of $h(x)$ have changed sign. The invariant point is (0, 1).

d) The graph of $g(x)$ is the reflection of the graph of $f(x)$ in the x-axis, while the graph of $h(x)$ is the reflection of the graph of $f(x)$ in the y-axis.

2. a)

x	$f(x) = x^2$	$g(x) = 3f(x)$	$h(x) = \frac{1}{3}f(x)$
−6	36	108	12
−3	9	27	3
0	0	0	0
3	9	27	3
6	36	108	12

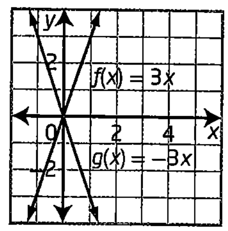
b)



c) The y-coordinates of $g(x)$ are three times larger. The invariant point is (0, 0). The y-coordinates of $h(x)$ are three times smaller. The invariant point is (0, 0).

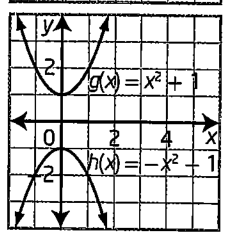
d) The graph of $g(x)$ is a vertical stretch by a factor of 3 of the graph of $f(x)$, while the graph of $h(x)$ is a vertical stretch by a factor of $\frac{1}{3}$ of the graph of $f(x)$.

3. a)



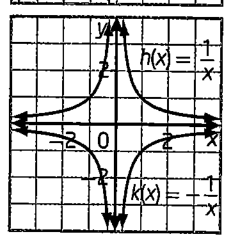
$g(x) = -3x$
$f(x)$: domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \in \mathrm{R}\}$
$g(x)$: domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \in \mathrm{R}\}$

b)



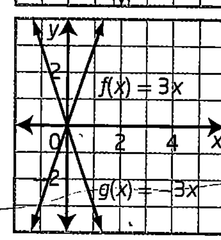
$h(x) = -x^2 - 1$
$g(x)$: domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \geq 1, y \in \mathrm{R}\}$
$h(x)$: domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \leq -1, y \in \mathrm{R}\}$

c)



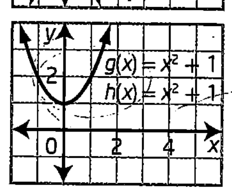
$k(x) = -\frac{1}{x}$
$h(x)$: domain $\{x \mid x \neq 0, x \in \mathrm{R}\}$, range $\{y \mid y \neq 0, y \in \mathrm{R}\}$
$k(x)$: domain $\{x \mid x \neq 0, x \in \mathrm{R}\}$, range $\{y \mid y \neq 0, y \in \mathrm{R}\}$

4. a)



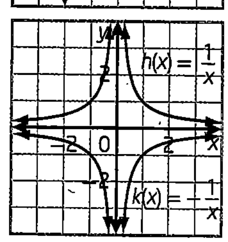
$g(x) = -3x$
$f(x)$: domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \in \mathrm{R}\}$
$g(x)$: domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \in \mathrm{R}\}$

b)



$h(x) = x^2 + 1$
$g(x)$: domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \geq 1, y \in \mathrm{R}\}$
$h(x)$: domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \geq 1, y \in \mathrm{R}\}$

c)



$k(x) = -\frac{1}{x}$
$h(x)$: domain $\{x \mid x \neq 0, x \in \mathrm{R}\}$, range $\{y \mid y \neq 0, y \in \mathrm{R}\}$
$k(x)$: domain $\{x \mid x \neq 0, x \in \mathrm{R}\}$, range $\{y \mid y \neq 0, y \in \mathrm{R}\}$

5. a) The graph of $y = 4f(x)$ is a vertical stretch by a factor of 4 of the graph of $y = f(x)$. $(x, y) \rightarrow (x, 4y)$

b) The graph of $y = f(3x)$ is a horizontal stretch by a factor of $\frac{1}{3}$ of the graph of $y = f(x)$. $(x, y) \rightarrow \left(\frac{x}{3}, y\right)$

c) The graph of $y = -f(x)$ is a reflection in the x-axis of the graph of $y = f(x)$. $(x, y) \rightarrow (x, -y)$

d) The graph of $y = f(-x)$ is a reflection in the y-axis of the graph of $y = f(x)$. $(x, y) \rightarrow (-x, y)$

6. a) domain $\{x \mid -6 \le x \le 6, x \in R\}$, range $\{y \mid -8 \le y \le 8, y \in R\}$

b) The vertical stretch affects the range by increasing it by the stretch factor of 2.

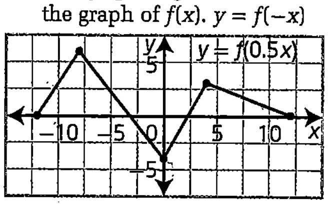
7. a) The graph of $g(x)$ is a vertical stretch by a factor of 4 of the graph of $f(x)$. $y = 4f(x)$

b) The graph of $g(x)$ is a reflection in the x-axis of the graph of $f(x)$. $y = -f(x)$

c) The graph of $g(x)$ is a horizontal stretch by a factor of $\frac{1}{3}$ of the graph of $f(x)$. $y = f(3x)$

d) The graph of $g(x)$ is a reflection in the y-axis of the graph of $f(x)$. $y = f(-x)$

8.



9. a) horizontally stretched by a factor of $\frac{1}{4}$

b) horizontally stretched by a factor of 4

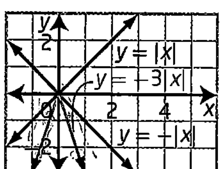
c) vertically stretched by a factor of $\frac{1}{2}$

d) vertically stretched by a factor of 4

e) horizontally stretched by a factor of $\frac{1}{3}$ and reflected in the y-axis

f) vertically stretched by a factor of 3 and reflected in the x-axis

10. a)



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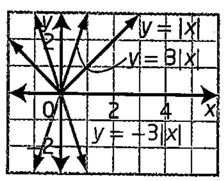
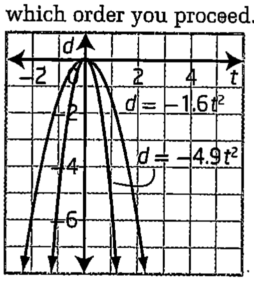


c) They are both incorrect. It does not matter in which order you proceed.

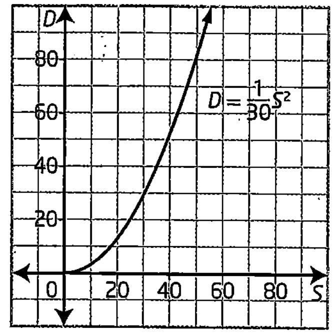
11. a)



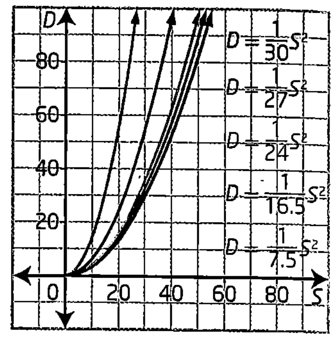
b) Both the functions are reflections of the base function in the t-axis. The object falling on Earth is stretched vertically more than the object falling on the moon.

12. Example: When the graph of $y = f(x)$ is transformed to the graph of $y = f(bx)$, it undergoes a horizontal stretch about the y-axis by a factor of $\frac{1}{|b|}$ and only the x-coordinates are affected. When the graph of $y = f(x)$ is transformed to the graph of $y = af(x)$, it undergoes a vertical stretch about the x-axis by a factor of $|a|$ and only the y-coordinates are affected.

13. a)



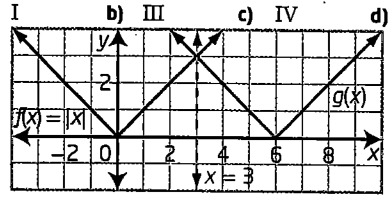
b) As the drag factor decreases, the length of the skid mark increases for the same speed.



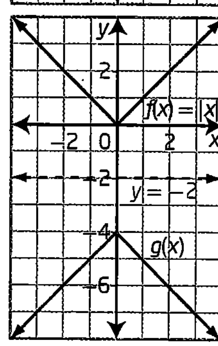
14. a) $x = -4, x = 3$ b) $x = 4, x = -3$ c) $x = -8, x = 6$ d) $x = -2, x = 1.5$

15. a) I b) III c) IV d) IV

16. a)



b)



C1 Example: When the input values for $g(x)$ are b times the input values for $f(x)$, the scale factor must be $\frac{1}{b}$ for the same output values. $g(x) = f\left(\frac{1}{b}(bx)\right) = f(x)$

C2 Examples:

a) a vertical stretch or a reflection in the x-axis

b) a horizontal stretch or a reflection in the y-axis

C3

$f(x)$	$g(x)$	Transformation
(5, 6)	(5, −6)	reflection in the x-axis
(4, 8)	(−4, 8)	reflection in the y-axis
(2, 3)	(2, 12)	vertical stretch by a factor of 4
(4, −12)	(2, −6)	horizontal stretch by a factor of $\frac{1}{2}$ and vertical stretch by a factor of $\frac{1}{2}$