

## 1.3 Rational Exponents and Radicals

### A. Rational Exponent Rules:

1)  $a^{\frac{1}{n}} = \sqrt[n]{a}$  where  $n$  is a natural number

2)  $a^{\frac{m}{n}} = \sqrt[n]{a^m}$  or  $a^{\frac{m}{n}} = (\sqrt[n]{a})^m$  where  $m$  and  $n$  are natural numbers

**Example 1:** Write as a radical and then evaluate.

$$\begin{aligned} \text{a) } 1000^{\frac{1}{3}} \\ &= \sqrt[3]{1000} \\ &= 10 \end{aligned}$$

$$\begin{aligned} \text{b) } 0.25^{\frac{1}{2}} \\ &= \sqrt{0.25} \\ &= 0.5 \end{aligned}$$

$$\begin{aligned} \text{c) } \left(\frac{16}{81}\right)^{\frac{1}{4}} \\ &= \sqrt[4]{\frac{16}{81}} \\ &= \frac{\sqrt[4]{16}}{\sqrt[4]{81}} \\ &= \frac{2}{3} \end{aligned}$$

$$\begin{aligned} \text{d) } (-64)^{\frac{1}{3}} \\ &= \sqrt[3]{-64} \\ &= -4 \end{aligned}$$

**Example 2:** Write in exponential form.

$$\begin{aligned} \text{a) } \sqrt{3^5} \\ &= 3^{\frac{5}{2}} \end{aligned}$$

$$\begin{aligned} \text{b) } (\sqrt[3]{25})^2 \\ &= 25^{\frac{2}{3}} \end{aligned}$$

$$\begin{aligned} \text{c) } \sqrt{\sqrt{x^4}} \\ &= ((x^4)^{\frac{1}{2}})^{\frac{1}{2}} \\ &= (x^2)^{\frac{1}{2}} \\ &= x \end{aligned}$$

$$\begin{aligned} \text{d) } \sqrt[5]{4x^4} \\ &= ((4x^4)^{\frac{1}{5}})^{\frac{1}{2}} \\ &= (4x^4)^{\frac{1}{10}} \end{aligned}$$

**Example 3:** Write in radical form and then evaluate.

$$\begin{aligned} \text{a) } 8^{\frac{2}{3}} \\ &= (\sqrt[3]{8})^2 \\ &= 2^2 \\ &= 4 \end{aligned}$$

$$\begin{aligned} \text{b) } 81^{\frac{3}{4}} \\ &= (\sqrt[4]{81})^3 \\ &= 3^3 \\ &= 27 \end{aligned}$$

$$\begin{aligned} \text{c) } 0.04^{\frac{3}{2}} \\ &= (\sqrt{0.04})^3 \\ &= (0.2)^3 \\ &= 0.008 \end{aligned}$$

$$\begin{aligned} \text{d) } (-32)^{0.4} &\rightarrow \frac{4}{10} = \frac{2}{5} \\ &= (\sqrt[5]{-32})^2 \\ &= (-2)^2 \\ &= 4 \end{aligned}$$

**Example 4:** Simplify each expression. Show your work (when necessary). Write your final answer with positive exponents.

$$\begin{aligned} \text{a) } (x^3y^2)(x^2y^{-\frac{3}{2}}) \\ = x^5y^{\frac{1}{2}} \end{aligned}$$

$$\begin{aligned} \text{b) } \frac{10a^{\frac{3}{2}}b^3}{2a^2b^{-2}} \\ = 5a^{-\frac{1}{2}}b^5 \\ = \frac{5b^5}{a^{\frac{1}{2}}} \end{aligned}$$

$$\begin{aligned} \text{c) } (32x^3)^{-\frac{3}{5}} \\ = \frac{1}{(32x^3)^{\frac{3}{5}}} \\ = \frac{1}{(\sqrt[5]{32})^3 x^{\frac{9}{5}}} \\ = \frac{1}{2^3 x^{\frac{9}{5}}} \\ = \frac{1}{8 x^{\frac{9}{5}}} \end{aligned}$$

$$\begin{aligned} \text{d) } \frac{25a^{\frac{1}{2}}}{100a^5b^{-\frac{1}{2}}} \\ = \frac{1 a^{-4} b^{\frac{1}{2}}}{4} \\ = \frac{b^{\frac{1}{2}}}{4a^4} \end{aligned}$$