1.3 Combining Transformations

General Transformation Equation:

$$y = af(b(x-h)) + k \qquad \text{or} \qquad y - k = af(b(x-h))$$

> a __vertical stretch by a factor of $|a|$__ ← absolute value
>   __if a is negative, reflection over x-axis__

> b __horizontal stretch by a factor of $|\frac{1}{b}|$__ ←
>   __if b is negative, reflection over y-axis__

> h __horizontal translation (left or right __ units)__

> k __vertical translation (up or down __ units)__

To simplify the procedure of combining transformations we perform transformations in the following order:

- ___stretches___ (a and b values)

- ___reflection___ over x/y axes ("−")

- ___translations___ (h and k values)

Example 1: Describe the transformations using an appropriate order to graph the new function from the original $y = f(x)$. Then, give the mapping for each.

Note: Functions of the form $y = f(bx + h)$ must be rewritten in ___$f(b(x + \frac{h}{b}))$___ form. ← factored form!

a) $y = -f(2(x+3)) - 1$

$b = 2 \rightarrow$ horizontal stretch by a factor of $\frac{1}{2}$

a is neg $\rightarrow$ reflection over x-axis

$h = -3 \rightarrow$ horizontal translation 3 units left

$k = -1 \rightarrow$ vertical translation 1 unit down.

$(x,y) \rightarrow (\frac{1}{2}x - 3, -y - 1)$

b) $y - 5 = 4f(-3x - 6)$ rewrite $y = 4f(-3(x+2)) + 5$
   $+5$ ↑ factor! $+5$

$|a| = 4 \rightarrow$ vertical stretch by a factor of 4

$|b| = 3 \rightarrow$ horizontal stretch by a factor of $\frac{1}{3}$

b is neg $\rightarrow$ reflection over y-axis

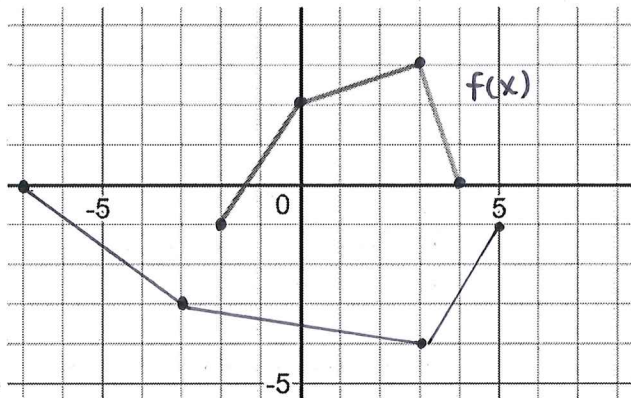$h = -2 \rightarrow$ horizontal translation 2 units left

$k = 5 \rightarrow$ vertical translation 5 units up

$(x,y) \rightarrow (-\frac{1}{3}x - 2, 4y + 5)$

Example 2: Given the graph of $y = f(x)$, describe the transformations in an appropriate order needed to sketch the new graph. Sketch the transformed graph.

$$y = -f\left(\tfrac{1}{2}(x+3)\right) - 1$$

- $|b| = \tfrac{1}{2} \rightarrow$ horiz. stretch by factor of 2

- a is neg $\rightarrow$ reflection over x-axis

- $h = -3 \rightarrow$ horiz. translation 3 units left

- $k = -1 \rightarrow$ vert. translation 1 unit down.

f(x)

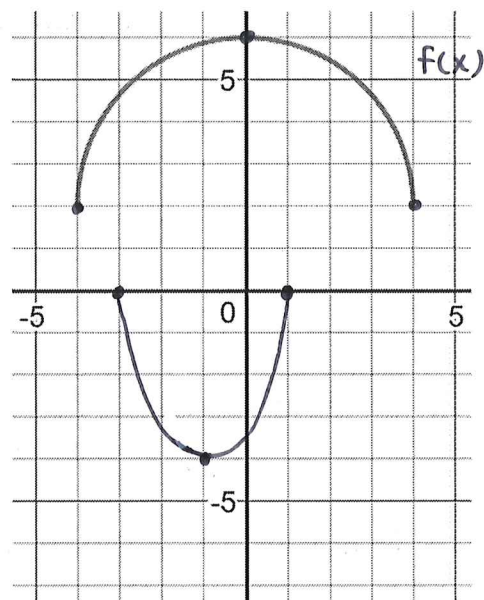| -3	×2	x		y	×-1	-1
-7	-4 ⟶2	-1		1	1	0
-3	0 0	2		-2	-3
3	6 3	3		-3	-4
5	8 4	0		0	-1

Example 3: Given the graph of $y = f(x)$, describe the transformations in an appropriate order needed to sketch the new graph. Sketch the transformed graph.

$$y - 2 = -f(-2x - 2)$$
rewrite

$$y = -f(-2(x+1)) + 2$$

- $|b| = 2 \rightarrow$ horiz. stretch by factor of $\tfrac{1}{2}$

- b is neg $\rightarrow$ reflection over y-axis

- a is neg $\rightarrow$ " " x-axis

- $h = -1 \rightarrow$ horiz. translation 1 unit left

- $k = 2 \rightarrow$ vert. translation 2 units up

f(x)

-1	×-1	×$\tfrac{1}{2}$	x		y	×-1	+2
1	1 -1 -4		1		1	-1	0
-1	0 0 0		6		6	-6	-4
-3	-1 1 4		1		1	-1	0

Example 4: The point $(4, -6)$ is on the graph of $y = f(x)$. What is the image point under each transformation of the graph of $f(x)$?

a) $y - 6 = 2f(2x - 4)$

    rewrite

$$y = 2f(2(x-2)) + 6$$

$$(x, y) \rightarrow (\tfrac{1}{2}x + 2, \; 2y + 6)$$

$$(4, -6) \rightarrow (\tfrac{1}{2}(4) + 2, \; 2(-6) + 6)$$

$$\rightarrow (2 + 2, \; -12 + 6)$$

$$\boxed{(4, -6) \rightarrow (4, -6)}$$

b) $y = -\tfrac{2}{3}f(x + 1) - 3$

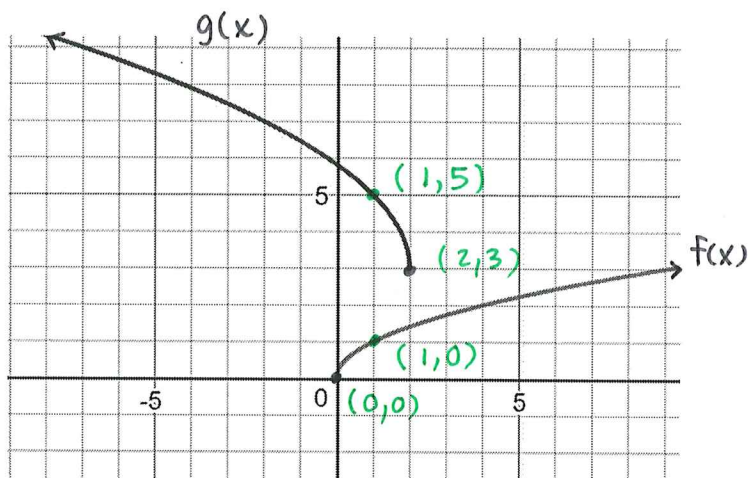$$(x, y) \rightarrow (x - 1, \; -\tfrac{2}{3}y - 3)$$

$$(4, -6) \rightarrow (4 - 1, \; -\tfrac{2}{3}(-6) - 3)$$

$$\rightarrow (3, \; 4 - 3)$$

$$\boxed{(4, -6) \rightarrow (3, 1)}$$

transformed

Example 5: The graph of $y = g(x)$ represents a transformation of the graph $y = f(x)$. Determine an equation for $g(x)$ in the form $y = af(b(x - h)) + k$.

- reflection over y-axis
  so b is negative
  (by observation)

- Use "start point" $(0, 0)$
  on original $f(x)$

  translated up 3 units
  so $k = 3$

  translated right 2 units right
  so $h = 2$

- vertical stretch by factor of 2
  so $a = 2$

  (compare similar points to find this)

g(x)

$(1, 5)$

$(2, 3)$

$(1, 0)$

$(0, 0)$

f(x)

$$g(x) = af(b(x - h)) + k$$

$$\boxed{g(x) = 2f(-(x - 2)) + 3}$$

Practice: p.38 #2, 5 – 10, 12