1.4 Inverses: Part I

The inverse of a relation can be found by ___interchanging___ the x and y coordinates of the original function.

$$(x, y) \rightarrow (\underline{y}, \underline{x})$$

For every (x, y) of a relation, there is an ordered pair (x, y) on the inverse of that relation.

Example 1:

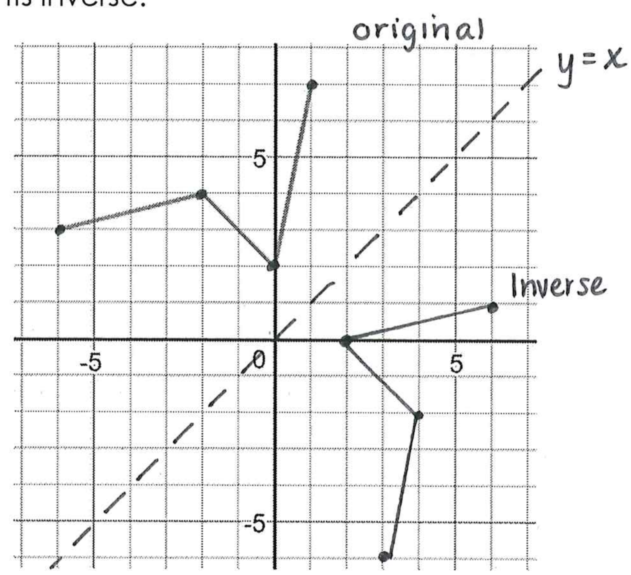
a) Given the graph of the relation below sketch the graph of its inverse.

x	y
-6	3
-2	4
0	2
1	7

original

x	y
3	-6
4	-2
2	0
7	1

inverse



b) On the above graph, sketch the line $y = x$. What do you notice about the graphs with respect to this line?

The graph of a relation and its inverse are ___reflections___ of each other in the line $y = x$.

c) Is the graph of the original relation a function? How do you know?

Yes, the original is a function. There is only one y-value for each x-value.

★ Can also check with vertical line test.

d) Is the graph of the inverse a function? How could you tell without graphing the inverse whether it would be a function?

No, the inverse is not a function. Some x-values have more than one y-value.

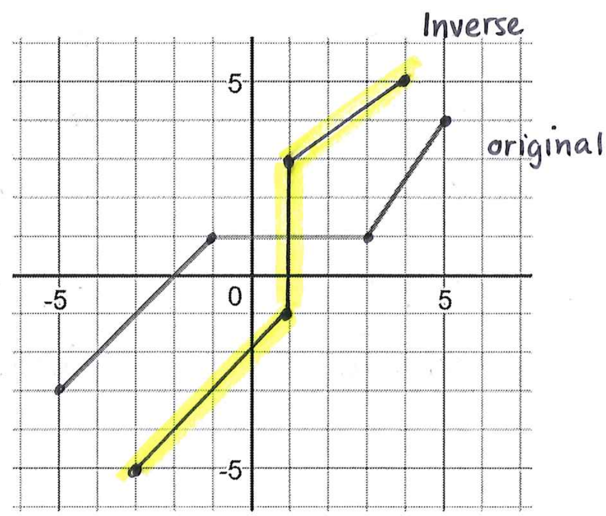
Horizontal Line Test:

- A test used to determine whether the graph of an inverse relation will be a function.
- If a horizontal line intersects an original graph in more than one place then the inverse of the relation is not a function.

The inverse of a function $y = f(x)$ may be written in the form $x = f(y)$.

When the inverse of a function is itself a function then we use the notation: $f^{-1}(x)$.
"f inverse of x"

Example 2: Consider the function $y = f(x)$ below:



a) Without graphing will the inverse graph be a function?

No (original fails the Horizontal line test)

b) Sketch the graph of $x = f(y)$
(inverse)

c) State the domain and range for both the original and the inverse.

original $y = f(x)$

D: $\{x \mid -5 \le x \le 5,\ x \in \mathbb{R}\}$

R: $\{y \mid -3 \le y \le 4,\ y \in \mathbb{R}\}$

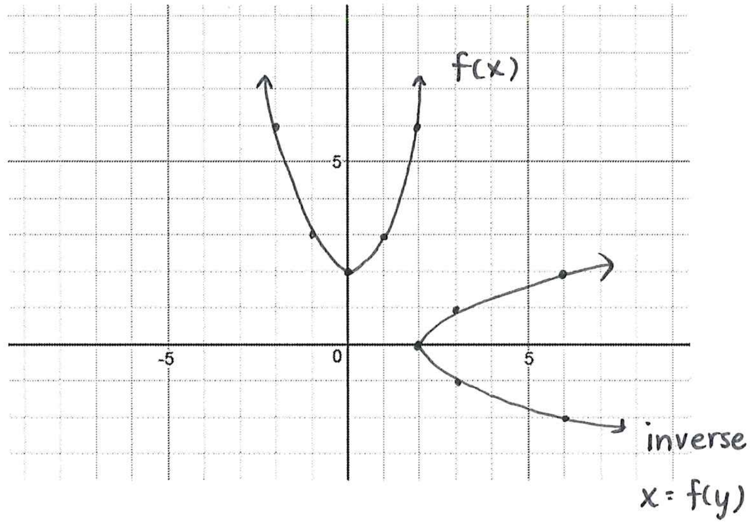
inverse $x = f(y)$

D: $\{x \mid -3 \le x \le 4,\ x \in \mathbb{R}\}$

R: $\{y \mid -5 \le y \le 5,\ y \in \mathbb{R}\}$

Example 3: Consider the function $f(x) = x^2 + 2$.

a) Graph the function $f(x)$. Is the inverse of $f(x)$ a function?



Not a function because f(x) fails Horizontal Line Test.

b) Graph the inverse of $f(x)$.

c) State the domain and range of $f(x)$ and its inverse.

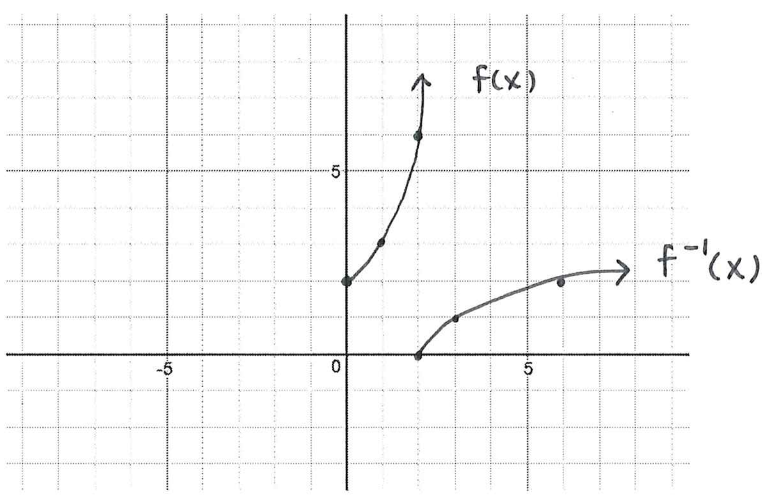
original $D: \{x \mid x \in \mathbb{R}\}$
$R: \{y \mid y \geq 2, y \in \mathbb{R}\}$

inverse $D: \{x \mid x \geq 2, x \in \mathbb{R}\}$
$R: \{y \mid y \in \mathbb{R}\}$

d) Restrict the domain of $f(x)$ so that its inverse will be a function.

if $x \geq 0$ we would only get the right side of the parabola } could have done $x \leq 0$ as well.

e) Sketch $f(x)$ with its restricted domain and its inverse.



} can use this notation now since the inverse is a function.

Practice: p.51 #1 – 4, 7, 8, 13ab