1.4 Inverses: Part 2

Example 1: Consider the function $f(x) = \frac{1}{2}x + 3$

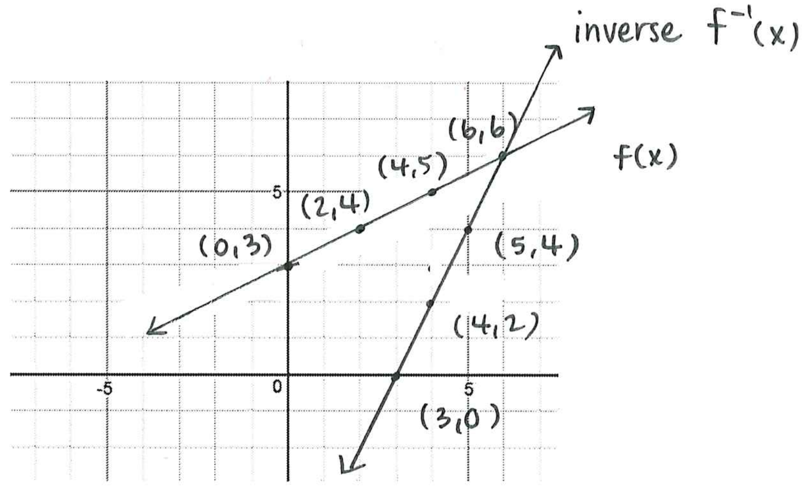
a) Graph the function $f(x)$ and its inverse.

$f(x) = \frac{1}{2}x + 3$

$y = mx + b$

$m = \text{slope} = \frac{\text{rise}}{\text{run}}$ $\qquad m = \frac{1}{2}$

$b = \text{y-intercept}$ $\qquad b = 3$



b) Find the equation of the inverse using the point-slope formula. $y - y_1 = m(x - x_1)$

$m = \frac{\text{rise}}{\text{run}} = \frac{2}{1}$ or 2

use point $(4, 2)$
$x_1 \; y_1$

$y - 2 = 2(x - 4)$

$y - 2 = 2x - 8$

$+2 \qquad\qquad +2$

$y = 2x - 6$ or $f^{-1}(x) = 2x - 6$

c) We found the graph of the inverse by switching the x and y coordinates. We can find the equation of the inverse by switching the x and y variables in the original relation.

Find the equation of the inverse.

original function: $f(x) = \frac{1}{2}x + 3$

$y = \frac{1}{2}x + 3$

inverse function: ① switch the x & y variables

$2\left(x = \frac{1}{2}y + 3\right)$

② solve for y

$2x = y + 6$

$-6 \qquad -6$

$2x - 6 = y$ or $f^{-1}(x) = 2x - 6$

Example 2: Find the equation of the inverse of $f(x) = 4x - 5$

original : $y = 4x - 5$

inverse : $x = 4y - 5$

$+5 \qquad\qquad +5$

$\frac{x}{4} + \frac{5}{4} = \frac{4y}{4}$

$\frac{1}{4}x + \frac{5}{4} = y$ or $f^{-1}(x) = \frac{1}{4}x + \frac{5}{4}$

Example 3: Find the inverse relation for $f(x) = x^2 - 3$.

original : $y = x^2 - 3$

inverse : $x = y^2 - 3$

$+3 \qquad +3$

$\sqrt{x+3} = \sqrt{y^2}$

$\pm\sqrt{x+3} = y$

or $y = \pm\sqrt{x+3}$

* this inverse is <u>not</u> a function.

Example 4: Find the inverse of $f(x) = \frac{3}{x+2}$; $\underbrace{x \neq -2}_{\text{non-permissible value (n.p.v.)}}$

original : $y = \dfrac{3}{x+2}$

inverse : $(y+2)x = \dfrac{3}{\cancel{(y+2)}}\cancel{(y+2)}$

$\dfrac{(y+2)\cancel{x}}{\cancel{x}} = \dfrac{3}{x}$

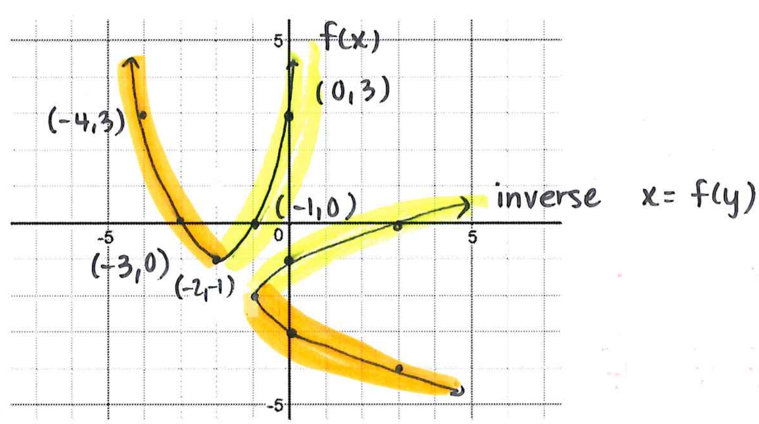
$y + 2 = \dfrac{3}{x}$

$-2 \qquad -2$

$y = \dfrac{3}{x} - 2;\ x \neq 0$

Example 5: Consider the function $f(x) = (x+2)^2 - 1$

a) Graph the function $f(x)$ and its inverse. Is the inverse a function? No !



b) Restrict the domain of $f(x)$ so that the inverse will be a function.

- If we only consider the right side of f(x), then only the top of the inverse is produced.

  restriction for f(x) would be $x \geq -2$

- If we only consider the left side of f(x), then only the bottom of the inverse is produced.

  restriction would be $x \leq -2$

c) Using the restriction on $f(x)$, find the equation of the inverse function.

equation of inverse

original: $y = (x+2)^2 - 1$

inverse: $x = (y+2)^2 - 1$

$+1 \qquad +1$

$\sqrt{x+1} = \sqrt{(y+2)^2}$

$\pm\sqrt{x+1} = y+2$

$-2 \qquad -2$

$\pm\sqrt{x+1} - 2 = y$

with restriction $x \geq -2$

$f^{-1}(x) = +\sqrt{x+1} - 2$

with restriction $x \leq -2$

$f^{-1}(x) = -\sqrt{x+1} - 2$

Practice: p.51 #5ace, 9, 10, 12ace, 14ac, 15ac