Pre-Calculus 12 : Year – End Review Booklet

Chapters 1 & 2 : Transformations

1. In what order should transformations be applied to a graph?

2. Describe the transformations in each equation in an appropriate order.

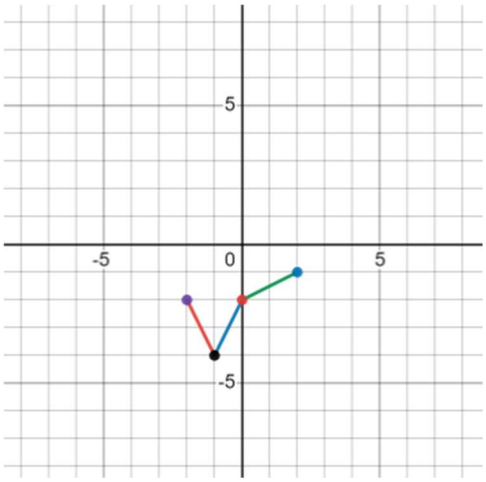
a) $2y - 8 = 6f(x - 2)$

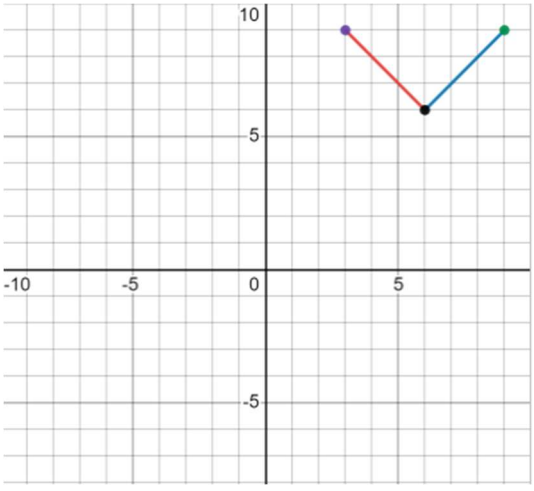
b) $y = -3f\big(-4(x - 1)\big) + 2$

3. Given the graph of $y = f(x)$, sketch the graph of the transformed function.

a) $y = f\left(-\frac{1}{4}x\right) + 1$

b) $f(x) = 2f(3x - 6) - 10$



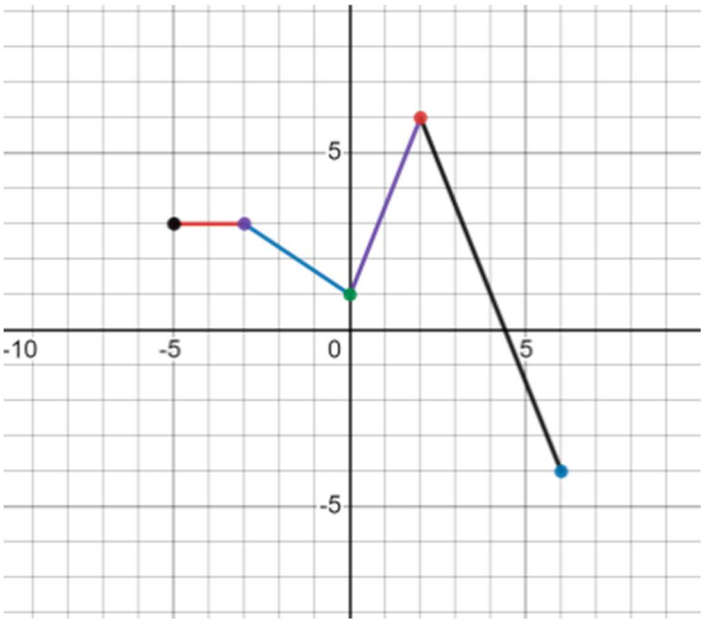


4. The following transformations are applied to a function $y = f(x)$.
   - Vertical stretch by a factor of 4
   - Horizontal stretch by a factor of 3
   - Reflection over the x – axis
   - Translated 2 units up, 5 units to the left

a) create a mapping notation for the transformations

b) If the point $(-2,5)$ is on $f(x)$, use the mapping notation to find the new point after the transformations are applied.

5. Sketch the inverse of the following function.

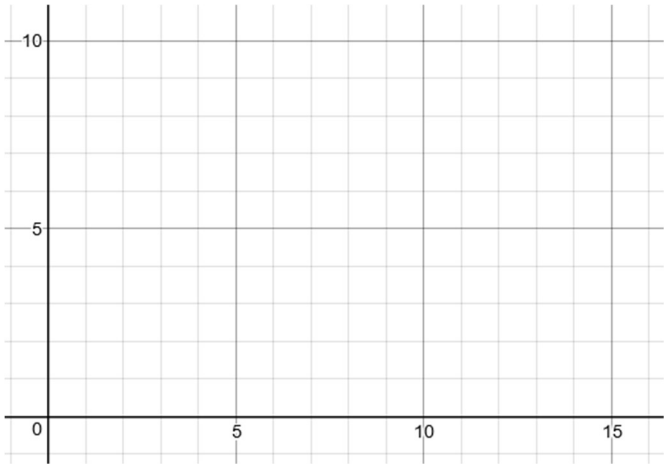


6. Find the inverse of $f(x) = \frac{3}{x-2}$

7. The domain and range of a function are $\{x|-3 \leq x \leq 6, x \in R\}$ and $\{y|y > 7, y \in R\}$. State the domain and range of the inverse function.

8. Sketch the graph of the function. State the domain and range.

$$y = 2\sqrt{x-3} + 4$$

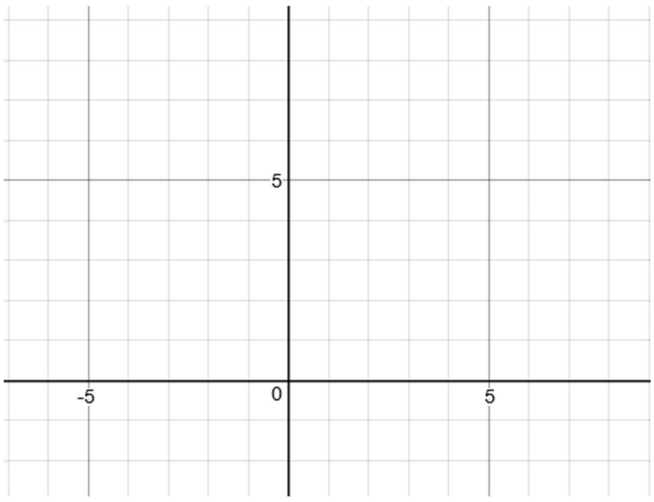


9. Write a single equation for a radical function with the given domain and range.

D: $\{x|x \geq 3, x \in R\}$
R: $\{y|y \leq -5, y \in R\}$

10. Solve the following equation graphically.

$$2\sqrt{x+2} = 1 - x$$



Chapter 3 : Polynomials

1. State the following for the given polynomial function: $f(x) = x^4 - 5x^3 + 2x^2 + 20x - 24$

Degree	
Type	
Sign of leading coefficient	
Constant term	
Maximum number of possible x - intercepts	
Value of the y - intercept	
End behavior	

2. Use the Factor Theorem to determine whether $x^4 - 2x^2 + 3x - 4$ has $x - 2$ as a factor.

3. Find the value of k if the remainder is 3 when $x^3 - x^2 + kx - 15$ is divided by $x - 2$.

4. For the following function, determine the x-intercepts, the degree and end behavior of the graph, the zeroes and their multiplicity, the y-intercept of the graph, intervals where the function is positive and negative.

$$f(x) = x^4 + 4x^3 - 7x^2 - 34x - 24$$

y – intercept

degree and end behavior

x – intercepts

zeroes and multiplicity

intervals of positive and negative

Chapter 4 : Trigonometry and the Unit Circle

1. Convert the given angle from radians to degrees or vice – versa.

a) $\frac{5\pi}{9}$

b) $240°$

2. Find one positive and one negative co – terminal angle for the original angles in question #1.

3. A circle has a central angle of $40°$ and a radius of $7\ ft$. Find the arclength of the sector.

4. A radius of a circle is $8\ cm$, and the length of an arc on the circle is $12\ cm$. In radians, what is the central angle that subtends this arc length?

5. The point $P(x, y)$ is located where the terminal arm of an angle θ and the unit circle intersect. Determine the coordinates of point P if:

a) $\theta = 210°$

b) $\theta = \frac{3\pi}{4}$

6. Identify a measure for the central angle θ in the interval $0 \leq \theta < 2\pi$ such that $P(\theta)$ is the given point.

a) $\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

b) $(1, -\sqrt{3})$

7. Solve $5\sin\theta + 2 = 1 + 3\sin\theta$; $0 \leq \theta < 2\pi$ Express your solution as an exact value.

Chapter 5 : Trigonometric Functions and Graphs

1. Determine the key features of the function $y = -5\sin\left(\frac{1}{2}\left(x - \frac{\pi}{2}\right)\right) + 15$

a) Amplitude

b) Period

c) Phase shift

d) Vertical displacement

e) Domain

f) Range

2. Write the equation of each sine function in the form $y = a \sin b(x - c) + d$ given its characteristics.

a) amplitude 2, period π, phase shift $\frac{\pi}{3}$ to the left, vertical displacement 1 unit down

b) amplitude $\frac{1}{4}$, period 6π, phase shift π to the right, vertical displacement 2 unit up

3. Graph the following function (show 2 full periods). State the period and phase shift.

$$y = 2\cos\frac{1}{2}\left(x - \frac{\pi}{2}\right) + 2$$

Period : _____ phase shift : _____

4. Solve the following trigonometric equations algebraically, using exact values.

a) $4\sin\left(x-\frac{\pi}{3}\right)=-2 \qquad 0\le x<2\pi$

b) $2\sin^2 x+5\sin x-3=0 \qquad 0\le x<2\pi$

Chapter 6 : Trigonometric Functions and Identities

1. Simplify the following :

a) $\cos(\theta+90°)$

b) $\sin 25°\cos 65°+\cos 25°\sin 65°$

2. Solve the following trigonometric equations; express your answers accurate to 2 decimal places for $0 \leq x < 2\pi$.

a) $2\sec^2 x + 5\sec x - 3 = 0$

b) $2\cos^2 x = -3\sin x$

3. Solve for all possible solutions in radians. Find a general solution.

$$\sin 2x = 2\sin x$$

4. Use sum or difference identities to find the exact value of each trigonometric expression.

a) $\sin 15°$

b) $\tan 165°$

5. Simplify the following:

a) $\cot^2 x \sin^2 x + \cos^2 x$

b) $\frac{\sec\theta - \cos\theta}{\csc\theta - \sin\theta}$

c) $(1 + \cos\theta)(\csc\theta - \cot\theta)$

6. Prove the identiy.

a) $\sin^3 x + \sin x \cos^2 x = \sin x$

b) $\frac{1+\cos x+\cos 2x}{\sin x+\sin 2x} = \cot x$

c) $\frac{\sin 2x}{2-2\cos^2 x} = \cot x$

d) $\frac{\cot x}{\csc x - 1} = \frac{\csc x + 1}{\cot x}$

Chapter 7 : Exponential Functions

1. Graph the base function $y = 2^x$ and the transformed function $y = -2(2)^{x-1} + 4$ on the same grid. Describe the transformations.

Transformations:

_____

_____

_____

_____

2. Solve.

a) $64^{4x} = 16^{x+5}$

b) $36^{-3n} \cdot 216 = \left(\frac{1}{216}\right)^{-2n}$

c) $\frac{9^{3x}}{243^{-x-1}} = 81^{2x}$

3. The half – life of sodium – 24 is 17 hours. A chemistry teacher has $40\ mg$ of sodium – 24. After how long will only $5\ mg$ remain?

4. A bacteria culture starts with 6250 bacteria and doubles every 3 hours. What was the population 9 hours ago?

5. At the initial count, there were 530 bacteria in a culture. Ten hours later, there were 14310 bacteria. What is the tripling period for this type of bacteria?

Chapter 8 : Logarithmic Functions

1. For the equation $y = 3\log_5(6(x+2)) - 4$, state:

a) Domain

b) Range

c) Equation of the asymptote

d) x – intercept (if it exists)

e) y – intercept (if it exists)

2. Simplify to a single log and then evaluate (if possible).

a) $2\log_2 12 - \left(\log_2 6 + \frac{1}{3}\log_2 27\right)$

b) $2\log_5 4 + \log_5 3 - \log_5 11$

c) $\log x - 3\log y + \frac{2}{3}\log z$

d) $\log_2(x+2) + \log_4 x$

3. Solve. Express your answer to the nearest hundredth, if necessary.

a) $\log_7(2x-3) - \log_7(x+2) = 1$

b) $\log_b(x+2) - \log_b 4 = \log_b 3x$

c) $2\log_4(x+4) - \log_4(x+12) = 1$

d) $2\ln(5x-2)=16$

4. Solve. Express your answer to the nearest hundredth, if necessary.

a) $9^{2x-1}=71^{x+2}$

b) $4(7^{x+2})=9^{2x-3}$

c) $e^{3x+1}=2$

Chapter 9 : Rational Functions

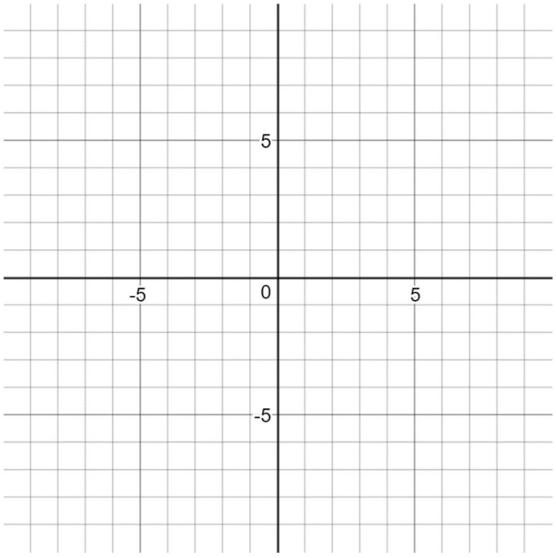
1. For each function, find the locations of any vertical asymptotes, points of discontinuity, and intercepts.

a) $y = \frac{x^2+4x}{x^2+9x+2}$

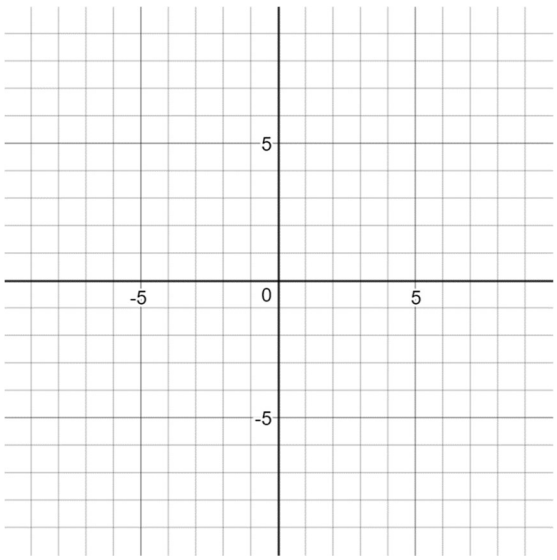
b) $y = \frac{2x^2-5x-3}{x^2-1}$

2. Graph the following functions using transformations and show at least 6 points. Label/identify any asymptotes.

a) $y = \frac{-2}{x+3} + 1$



b) $y = \frac{4x-5}{x-2}$



Chapter 10 : Composite Functions

1. If $f(x) = \sqrt{x+2}$ and $g(x) = |2x|$; find $f \circ g(-7)$

2. If $f(x) = x^2 + 7$ and $g(x) = 2x - 1$; find $f\big(g(x)\big)$

Chapter 12 : Geometric Sequences and Series

1. How many terms are in the geometric sequence $2, 6, 18, \ldots, 486$

2. The sum of an infinite series is 63 and the first term is 21. Find the common ratio.

3. Find the sum of the first 12 terms of the following geometric series: $12 + 4 + \frac{4}{3} + \ldots..$