

Pre-Calculus 12 : Year – End Review Booklet

Chapters 1 & 2 : Transformations

1. In what order should transformations be applied to a graph?

- reflections
- vertical / horizontal stretches
- vertical / horizontal translations } always done last

2. Describe the transformations in each equation in an appropriate order.

a) $2y - 8 = 6f(x - 2)$ rewrite

$$y = 3f(x-2) + 4$$

- 1) vert. stretch by a factor of 3
- 2) horiz. trans. 2 units right
- 3) vert. trans. 4 units up

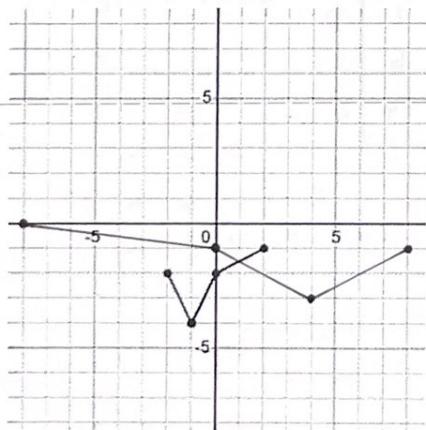
b) $y = -3f(-4(x - 1)) + 2$

- 1) reflection over x-axis
- 2) reflection over y-axis
- 3) vert. stretch by a factor of 3
- 4) horiz. stretch by a factor of $\frac{1}{4}$
- 5) horiz. trans. 1 unit right
- 6) vert. trans. 2 units up

3. Given the graph of $y = f(x)$, sketch the graph of the transformed function.

a) $y = f\left(-\frac{1}{4}x\right) + 1$

x	y
-8	2
-4	1
0	0
4	1
8	2

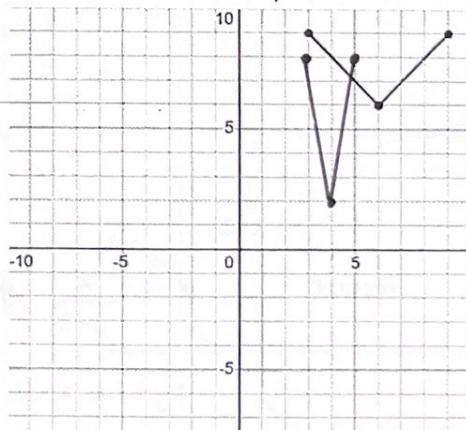


b) $f(x) = 2f(3(x-6)) - 10$

x	y
3	8
4	18
5	8

$$= 2f(3(x-2)) - 10$$

x	y
3	8
4	18
5	8



4. The following transformations are applied to a function $y = f(x)$.

- Vertical stretch by a factor of 4 $a = 4$
- Horizontal stretch by a factor of 3 $b = \frac{1}{3}$
- Reflection over the x -axis a is neg.
- Translated 2 units up, 5 units to the left
 $K = 2$ $h = -5$

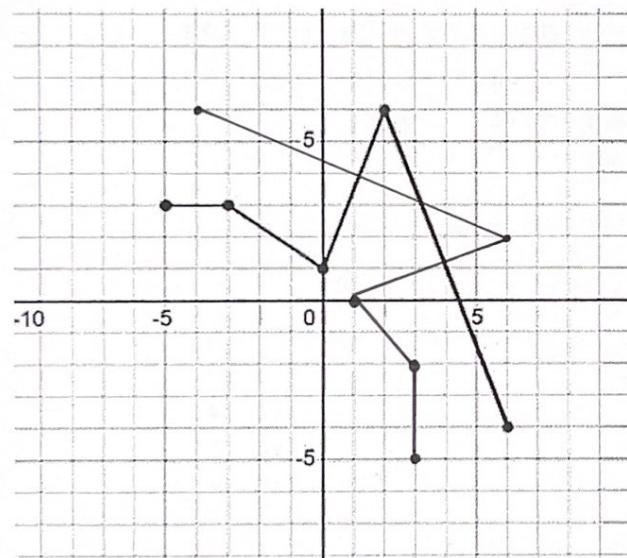
a) create a mapping notation for the transformations

$$(x, y) \longrightarrow (3x - 5, -4y + 2)$$

b) If the point $(-2, 5)$ is on $f(x)$, use the mapping notation to find the new point after the transformations are applied.

$$(-2, 5) \longrightarrow (3(-2) - 5, -4(5) + 2) \rightarrow (-6 - 5, -20 + 2) \\ \rightarrow (-11, -18)$$

5. Sketch the inverse of the following function.



x	y
-5	3
-3	3
0	1
2	6
6	-4

inverse

x	y
3	-5
3	-3
1	0
6	2
-4	6

6. Find the inverse of $f(x) = \frac{3}{x-2}$

$$y = \frac{3}{x-2}$$

inverse: $x^{(y-2)} = \frac{3}{y^2}$

$$xy - 2x = 3$$

$$\frac{xy}{x} = \frac{3+2x}{x} \quad y = \frac{3+2x}{x} \quad \text{or} \quad y = \frac{3}{x} + 2$$

7. The domain and range of a function are $\{x | -3 \leq x \leq 6, x \in \mathbb{R}\}$ and $\{y | y > 7, y \in \mathbb{R}\}$. State the domain and range of the inverse function.

inverse $D : \{x | x > 7, x \in \mathbb{R}\}$
 $R : \{y | -3 \leq y \leq 6, y \in \mathbb{R}\}$

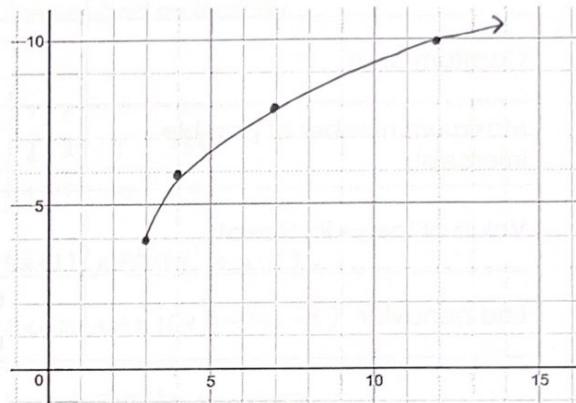
8. Sketch the graph of the function. State the domain and range.

$$y = 2\sqrt{x-3} + 4$$

+3	x	y	+2	+4
3	0	0	0	4
4	+	+	2	6
7	+	2	4	8
12	+	3	6	10

$$D: \{x | x \geq 3, x \in \mathbb{R}\}$$

$$R: \{y | y \geq 4, y \in \mathbb{R}\}$$

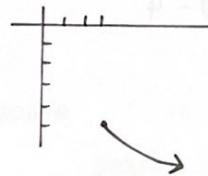


9. Write a single equation for a radical function with the given domain and range.

$$D: \{x | x \geq 3, x \in \mathbb{R}\}$$

$$R: \{y | y \leq -5, y \in \mathbb{R}\}$$

↑
reflection over
x-axis



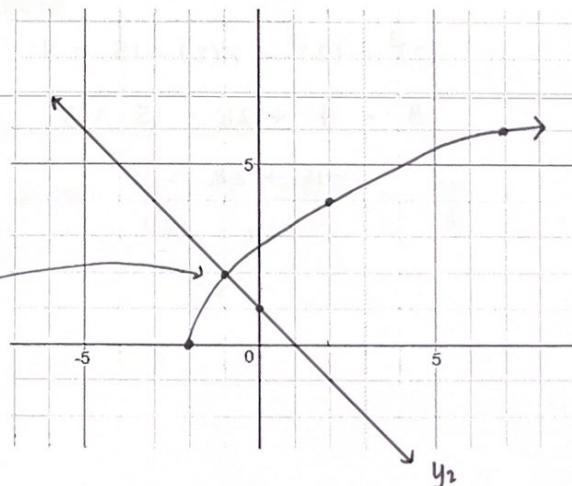
$$y = -\sqrt{x-3} - 5$$

10. Solve the following equation graphically.

$$y_1 = 2\sqrt{x+2} \quad y_2 = 1-x$$

-2	x	y	+2
-2	0	0	0
-1	+	2	2
2	+	2	4
7	+	3	6

solution
 $x = -1$



Chapter 3 : Polynomials

1. State the following for the given polynomial function: $f(x) = x^4 - 5x^3 + 2x^2 + 20x - 24$

Degree	4^{th}
Type	quartic
Sign of leading coefficient	positive
Constant term	-24
Maximum number of possible x -intercepts	4
Value of the y -intercept	(0, -24)
End behavior	up in quad II up in quad I

2. Use the Factor Theorem to determine whether $x^4 - 2x^2 + 3x - 4$ has $x - 2$ as a factor.

$$f(2) = (2)^4 - 2(2)^2 + 3(2) - 4$$

$$= 16 - 8 + 6 - 4$$

$$= 10$$

↑
remainder is not zero

so $x - 2$ is not a factor

3. Find the value of k if the remainder is 3 when $x^3 - x^2 + kx - 15$ is divided by $x - 2$.

$$f(2) = 3$$

$$(2)^3 - (2)^2 + k(2) - 15 = 3$$

$$8 - 4 + 2k - 15 = 3$$

$$-11 + 2k = 3$$

$$2k = 14$$

$$k = 7$$

4. For the following function, determine the x -intercepts, the degree and end behavior of the graph, the zeroes and their multiplicity, the y -intercept of the graph, intervals where the function is positive and negative.

$$f(x) = x^4 + 4x^3 - 7x^2 - 34x - 24$$

y -intercept

$$(0, -24)$$

degree and end behavior

4^{th} ; up in quad II, up in quad I

x -intercepts

$$-24: \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12, \pm 24$$

$$\begin{aligned} \text{try } x = -1 &: (-1)^4 + 4(-1)^3 - 7(-1)^2 - 34(-1) - 24 \\ &1 - 4 - 7 + 34 - 24 = 0 \end{aligned}$$

$x+1$ is a factor

$$\begin{array}{r} 1 \mid 1 & 4 & -7 & -34 & -24 \\ -\downarrow & 1 & 3 & -10 & -24 \\ \hline x \mid 1 & 3 & -10 & -24 & 0 \end{array}$$

$$(x+1)(x^3 + 3x^2 - 10x - 24)$$

$$\begin{aligned} \text{try } x = -2 &: (-2)^4 + 4(-2)^3 - 7(-2)^2 - 34(-2) - 24 \\ &-8 + 12 + 20 - 24 = 0 \checkmark \end{aligned}$$

$x+2$ is a factor

zeroes and multiplicity

$$\begin{array}{r} 2 \mid 1 & 3 & -10 & -24 \\ -\downarrow & 2 & 2 & -24 \\ \hline x \mid 1 & 1 & -12 & 0 \end{array}$$

$$(x+1)(x+2)(x^2 + x - 12)$$

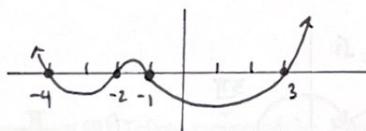
$$(x+1)(x+2)(x+4)(x-3)$$

x -intercepts / zeroes

$$x = -1, -2, -4, 3$$

each has a multiplicity of one

intervals of positive and negative



positive: $x < -4, -2 < x < -1, x > 3$

negative: $-4 < x < -2, -1 < x < 3$

Chapter 4 : Trigonometry and the Unit Circle

1. Convert the given angle from radians to degrees or vice-versa.

$$\text{a) } \frac{5\pi}{9} \cdot \frac{180}{\pi} = \frac{900}{9} = 100^\circ$$

$$\text{b) } 240^\circ \cdot \frac{\pi}{180} = \frac{240\pi}{180} = \frac{4\pi}{3}$$

2. Find one positive and one negative co-terminal angle for the original angles in question #1.

$$a) \frac{5\pi}{9} + 2\pi = \frac{5\pi}{9} + \frac{18\pi}{9} = \frac{23\pi}{9}$$

$$b) 240^\circ + 360^\circ = 600^\circ$$

$$\frac{5\pi}{9} - 2\pi = \frac{5\pi}{9} - \frac{18\pi}{9} = -\frac{13\pi}{9}$$

$$240^\circ - 360^\circ = -120^\circ$$

3. A circle has a central angle of 40° and a radius of 7 ft . Find the arclength of the sector.

$$\theta = 40^\circ$$

$$r = 7$$

$$x = ?$$

$$x = r\theta \cdot \frac{\pi}{180^\circ} = \frac{(7)(40)\pi}{180^\circ} = \frac{240\pi}{180} = \frac{14\pi}{9} \text{ ft.}$$

4. A radius of a circle is 8 cm , and the length of an arc on the circle is 12 cm . In radians, what is the central angle that subtends this arc length?

$$r = 8 \text{ cm} \quad x = 12 \text{ cm}$$

$$\theta = ?$$

$$x = r\theta$$

$$12 = 8\theta$$

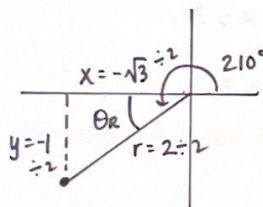
$$\frac{12}{8} = \theta$$

$$\theta = \frac{3}{2} \text{ rad.}$$

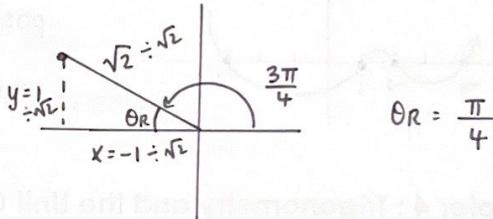
5. The point $P(x, y)$ is located where the terminal arm of an angle θ and the unit circle intersect. Determine the coordinates of point P if:

$$r = 1$$

$$a) \theta = 210^\circ$$



$$b) \theta = \frac{3\pi}{4}$$

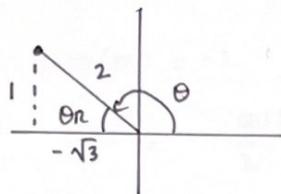


$$(x, y) = \left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$$

$$(x, y) = \left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$$

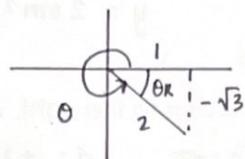
6. Identify a measure for the central angle θ in the interval $0 \leq \theta < 2\pi$ such that $P(\theta)$ is the given point.

a) $(-\frac{\sqrt{3}}{2}, \frac{1}{2})$



$$\theta_R = \frac{\pi}{6}$$

b) $(1, -\sqrt{3})$



$$\theta_R = \frac{\pi}{3}$$

$$\theta = 2\pi - \frac{\pi}{3}$$

$$\theta = \frac{5\pi}{3}$$

7. Solve $5 \sin \theta + 2 = 1 + 3 \sin \theta$; $0 \leq \theta < 2\pi$. Express your solution as an exact value.

$$2 \sin \theta = -1$$

$$\sin \theta = -\frac{1}{2}$$

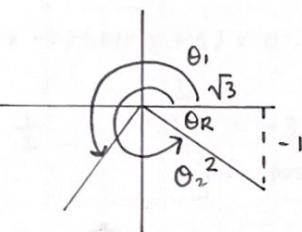
$$\theta_R = \frac{\pi}{6}$$

$$\theta_1 = \pi + \frac{\pi}{6}$$

$$\theta_2 = 2\pi - \frac{\pi}{6}$$

$$\theta_1 = \frac{7\pi}{6}$$

$$\theta_2 = \frac{11\pi}{6}$$



Chapter 5 : Trigonometric Functions and Graphs

1. Determine the key features of the function $y = -5 \sin\left(\frac{1}{2}\left(x - \frac{\pi}{2}\right)\right) + 15$

a) Amplitude $|-5| = 5$

b) Period $\frac{2\pi}{(1/2)} = 4\pi$

c) Phase shift $\frac{\pi}{2}$ to the right

d) Vertical displacement up 15

e) Domain $\{x | x \in \mathbb{R}\}$

f) Range $\{y | 10 \leq y \leq 20, y \in \mathbb{R}\}$

\uparrow
 $15 - 5$
amp
 $15 + 5$
 \uparrow

2. Write the equation of each sine function in the form $y = a \sin b(x - c) + d$ given its characteristics.

a) amplitude 2, period π , phase shift $\frac{\pi}{3}$ to the left, vertical displacement 1 unit down

$$a = 2 \quad \pi = \frac{2\pi}{b} \quad c = -\frac{\pi}{3} \quad d = -1$$

$$b = 2$$

$$y = 2 \sin 2(x + \frac{\pi}{3}) - 1$$

b) amplitude $\frac{1}{4}$, period 6π , phase shift π to the right, vertical displacement 2 unit up

$$a = \frac{1}{4} \quad 6\pi = \frac{2\pi}{b} \quad c = \pi \quad d = 2$$

$$b = \frac{1}{3}$$

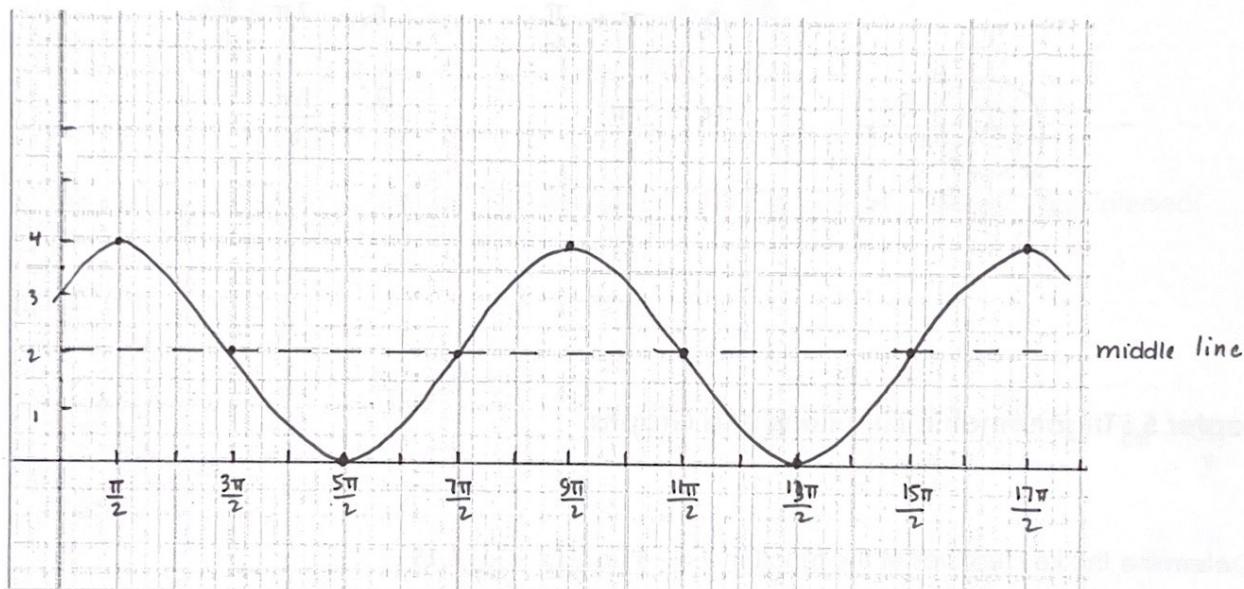
$$y = \frac{1}{4} \sin \frac{1}{3}(x - \pi) + 2$$

3. Graph the following function (show 2 full periods). State the period and phase shift.

$$y = 2 \cos \frac{1}{2}\left(x - \frac{\pi}{2}\right) + 2$$

$$\text{Period: } \frac{2\pi}{(\frac{1}{2})} = 4\pi$$

$$\text{phase shift: } \frac{\pi}{2} \text{ to the right}$$



x	$y = 2 \cos \frac{1}{2}(x - \frac{\pi}{2}) + 2$
$\frac{\pi}{2}$	4
$\frac{3\pi}{2}$	2
$\frac{5\pi}{2}$	0
$\frac{7\pi}{2}$	-2
$\frac{9\pi}{2}$	0
$\frac{11\pi}{2}$	2
$\frac{13\pi}{2}$	4
$\frac{15\pi}{2}$	2
$\frac{17\pi}{2}$	0

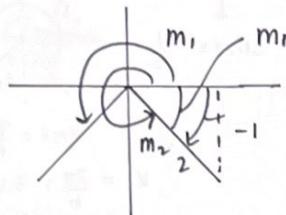
4. Solve the following trigonometric equations algebraically, using exact values.

a) $4 \sin\left(x - \frac{\pi}{3}\right) = -2 \quad 0 \leq x < 2\pi$

let $m = x - \frac{\pi}{3}$

$4 \sin(m) = -2$

$\sin(m) = -\frac{1}{2}$



$m_{ref} = \frac{\pi}{6}$

$m_1 = \frac{7\pi}{6}$
 $m_2 = \frac{11\pi}{6}$

b) $2 \sin^2 x + 5 \sin x - 3 = 0 \quad 0 \leq x < 2\pi$

$2 \sin^2 x - \sin x + 6 \sin x - 3 = 0$

$\sin x (2 \sin x - 1) + 3(2 \sin x - 1) = 0$

$(2 \sin x - 1)(\sin x + 3) = 0$

\downarrow
 $\sin x = \frac{1}{2}$

\downarrow
 $\sin x = -3$
not possible

$x = m + \frac{\pi}{3}$

$x_1 = m_1 + \frac{\pi}{3}$

$x_2 = m_2 + \frac{\pi}{3}$

$= \frac{7\pi}{6} + \frac{2\pi}{3}$

$= \frac{11\pi}{6} + \frac{2\pi}{3}$

$= \frac{9\pi}{6}$

$= \frac{13\pi}{6}$

$= \frac{3\pi}{2}$

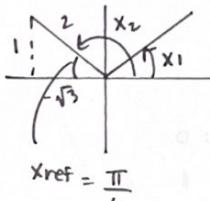
bigger than 2π
(our restriction)

Find coterminal angle.

$x = \frac{13\pi}{6} - 2\pi$

$x_3 = \frac{\pi}{6}$

$\sin x = \frac{1}{2}$



$x_1 = \frac{\pi}{6}$

$x_2 = \frac{5\pi}{6}$

Chapter 6 : Trigonometric Functions and Identities

1. Simplify the following :

a) $\cos(\theta + 90^\circ)$

$= \cos \theta \cos 90^\circ - \sin \theta \sin 90^\circ$

$= \cos \theta (0) - \sin \theta (1)$

$= -\sin \theta$

b) $\sin 25^\circ \cos 65^\circ + \cos 25^\circ \sin 65^\circ$

$= \sin(25^\circ + 65^\circ)$

$= \sin(90^\circ)$

$= 1$

2. Solve the following trigonometric equations; express your answers accurate to 2 decimal places for $0 \leq x < 2\pi$.

a) $2\sec^2 x + 5\sec x - 3 = 0$

$$2\sec^2 x - \sec x + 6\sec x - 3 = 0$$

$$\sec x (2\sec x - 1) + 3(2\sec x - 1) = 0$$

$$(2\sec x - 1)(\sec x + 3) = 0$$

$$\downarrow \sec x = \frac{1}{2}$$

$$\downarrow \sec x = -3$$

$\cos x = 2$

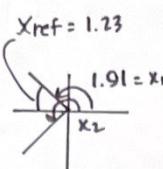
$$\cos x = -\frac{1}{3}$$

Not possible

use calc.

$$x_1 = 1.91$$

$$x_2 = 4.37$$



b) $2\cos^2 x = -3\sin x$

$$2(1-\sin^2 x) = -3\sin x$$

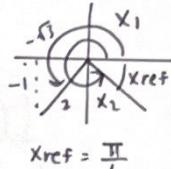
$$0 = 2\sin^2 x - 3\sin x - 2$$

$$0 = 2\sin x(\sin x - 2) + 1(\sin x - 2)$$

$$0 = (\sin x - 2)(2\sin x + 1)$$

$$\downarrow \sin x = 2 \quad \downarrow \sin x = -\frac{1}{2}$$

not possible



$$x_1 = \frac{7\pi}{6} = 3.67$$

$$x_2 = \frac{11\pi}{6} = 5.76$$

3. Solve for all possible solutions in radians. Find a general solution.

$$\sin 2x = 2\sin x$$

$$\sin 2x - 2\sin x = 0$$

$$2\sin x \cos x - 2\sin x = 0$$

$$2\sin x(\cos x - 1) = 0$$

$$\downarrow$$

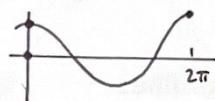
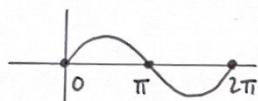
$$\downarrow$$

$$2\sin x = 0$$

$$\cos x - 1 = 0$$

$$\sin x = 0$$

$$\cos x = 1$$



$$x = \pi n, n \in \mathbb{Z}$$

4. Use sum or difference identities to find the exact value of each trigonometric expression.

a) $\sin 15^\circ$

$$= \sin(45^\circ - 30^\circ)$$

$$= \sin 45^\circ \cos 30^\circ - \cos 45^\circ \sin 30^\circ$$

$$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)$$

$$= \frac{\sqrt{3} - 1}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{\sqrt{6} - \sqrt{2}}{2\sqrt{4}}$$

$$= \frac{\sqrt{6} - \sqrt{2}}{4}$$

b) $\tan 165^\circ$

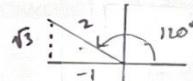
$$= \tan(120^\circ + 45^\circ)$$

$$= \frac{\tan 120^\circ + \tan 45^\circ}{1 - \tan 120^\circ \tan 45^\circ}$$

$$= \frac{\frac{\sqrt{3}}{-1} + 1}{1 - (-\sqrt{3})(1)}$$

$$= \frac{(-\sqrt{3} + 1)(1 - \sqrt{3})}{1 + \sqrt{3}(1 - \sqrt{3})}$$

$$= \frac{-\sqrt{3} + \sqrt{9} - \sqrt{3} + 1}{1 - \sqrt{9}}$$



$$= \frac{4 - 2\sqrt{3}}{-2}$$

$$= -2 + \sqrt{3}$$

5. Simplify the following:

a) $\cot^2 x \sin^2 x + \cos^2 x$

$$= \frac{\cos^2 x}{\sin^2 x} \cdot \frac{\sin^2 x}{\sin^2 x} + \cos^2 x$$

$$= \cos^2 x + \cos^2 x$$

$$= 2\cos^2 x$$

b) $\frac{\sec \theta - \cos \theta}{\csc \theta - \sin \theta}$

$$= \frac{\frac{1}{\cos \theta} - \cos \theta \cdot \frac{\cos \theta}{\cos \theta}}{\frac{1}{\sin \theta} - \sin \theta \cdot \frac{\sin \theta}{\sin \theta}}$$

$$= \frac{\frac{1 - \cos^2 \theta}{\cos \theta}}{\frac{1 - \sin^2 \theta}{\sin \theta}}$$

$$= \frac{\sin^2 \theta}{\cos \theta} \cdot \frac{\sin \theta}{\cos^2 \theta}$$

$$= \frac{\sin^3 \theta}{\cos^3 \theta}$$

$$= \tan^3 \theta$$

c) $(1 + \cos \theta)(\csc \theta - \cot \theta)$

$$= \csc \theta - \cot \theta + \cos \theta \csc \theta - \cos \theta \cot \theta$$

$$= \frac{1}{\sin \theta} - \frac{\cos \theta}{\sin \theta} + \cos \theta \cdot \cancel{\frac{1}{\sin \theta}} - \cos \theta \cdot \frac{\cos \theta}{\sin \theta}$$

$$= \frac{1 - \cos^2 \theta}{\sin \theta}$$

$$= \frac{\sin^2 \theta}{\sin \theta} = \sin \theta$$

6. Prove the identity.

a) $\sin^3 x + \sin x \cos^2 x = \sin x$

$$\begin{array}{r|l} \sin x (\sin^2 x + \cos^2 x) & \sin x \\ \hline \sin x (1) & \\ \hline \sin x & \end{array}$$

b) $\frac{1 + \cos x + \cos 2x}{\sin x + \sin 2x} = \cot x$

$$\begin{array}{r|l} \frac{x + \cos x + (2\cos^2 x - 1)}{\sin x + 2\sin x \cos x} & \cot x \\ \hline \end{array}$$

$$\begin{array}{r|l} \frac{\cos x (1 + 2\cos x)}{\sin x (1 + 2\cos x)} & \\ \hline \end{array}$$

$$\begin{array}{r|l} \frac{\cos x}{\sin x} & \\ \hline \end{array}$$

$$\begin{array}{r|l} \cot x & \\ \hline \end{array}$$

$$c) \frac{\sin 2x}{2-2\cos^2 x} = \cot x$$

$$\begin{array}{c|c} \frac{2\sin x \cos x}{2(1-\cos^2 x)} & \cot x \\ \hline \frac{\sin x \cos x}{\sin^2 x} & \\ \hline \frac{\cos x}{\sin x} & \\ \hline \cot x & \end{array}$$

$$d) \frac{\cot x}{\csc x - 1} = \frac{\csc x + 1}{\cot x}$$

$$\begin{array}{c|c} \frac{\cot x (\csc x + 1)}{(\csc x - 1)(\csc x + 1)} & \frac{\csc x + 1}{\cot x} \\ \hline \frac{\cot x (\csc x + 1)}{\csc^2 x - 1} & \\ \hline \frac{\cot x (\csc x + 1)}{\cot^2 x} & \\ \hline \frac{\csc x + 1}{\cot x} & \end{array}$$

Chapter 7 : Exponential Functions

1. Graph the base function $y = 2^x$ and the transformed function $y = -2(2)^{x-1} + 4$ on the same grid. Describe the transformations.

Transformations:

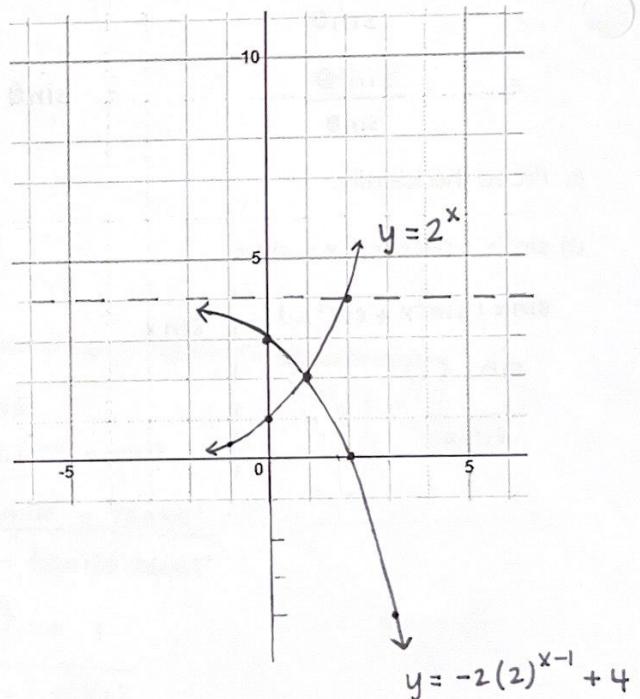
reflection over x-axis

vertical stretch by factor of 2

horiz. trans. 1 unit right

vert. trans. 4 units up

+1	x	y	-2	+4
0	\cancel{x}	\cancel{y}	-2	3
1	\cancel{x}	\cancel{y}	-2	2
2	\cancel{x}	\cancel{y}	-2	0
3	\cancel{x}	\cancel{y}	-2	-4



2. Solve.

a) $64^{4x} = 16^{x+5}$

$$4^{3(4x)} = 4^{2(x+5)}$$

exp. only : $12x = 2x + 10$

$$10x = 10$$

$$\boxed{x = 1}$$

b) $36^{-3n} \cdot 216 = \left(\frac{1}{216}\right)^{-2n}$

$$b^{2(-3n)} \cdot b^3 = b^{-3(-2n)}$$

exp. only : $-6n + 3 = 6n$

$$3 = 12n$$

$$\boxed{\frac{1}{4} = n}$$

c) $\frac{9^{3x}}{243^{-x-1}} = 81^{2x}$

$$\frac{3^{2(3x)}}{3^{5(-x-1)}} = 3^{4(2x)}$$

exp. only : $6x - 5(-x-1) = 8x$

$$6x + 5x + 5 = 8x$$

$$3x = -5$$

$$\boxed{x = -\frac{5}{3}}$$

3. The half-life of sodium-24 is 17 hours. A chemistry teacher has 40 mg of sodium-24. After how long will only 5 mg remain?

$$A = 5$$

$$A_0 = 40$$

$$t = ?$$

$$T = 17$$

$$C = \frac{1}{2}$$

$$A = A_0(C)^{\frac{t}{T}}$$

$$(\frac{1}{2})^3 = (\frac{1}{2})^{\frac{t}{17}}$$

$$\frac{5}{40} = \frac{40}{40}(\frac{1}{2})^{\frac{t}{17}}$$

exp. only : $3 = \frac{t}{17}$

$$\boxed{t = 51 \text{ hr}}$$

4. A bacteria culture starts with 6250 bacteria and doubles every 3 hours. What was the population 9 hours ago?

$$A = 6250$$

$$6250 = A_0(2)^{\frac{9}{3}}$$

$$A_0 = ?$$

$$6250 = A_0(2)^3$$

$$t = 9$$

$$T = 3$$

$$C = 2$$

$$6250 = 8A_0$$

$$781.25 = A_0$$

$\boxed{781 \text{ bacteria}}$

- c) 5. At the initial count, there were 530 bacteria in a culture. Ten hours later, there were 14310 bacteria. What is the tripling period for this type of bacteria?

$$A_0 = 530$$

$$A = 14310$$

$$t = 10$$

$$T = ?$$

$$C = 3$$

$$\frac{14310}{530} = \frac{530(3)^{\frac{10}{T}}}{530}$$

$$27 = 3^{\frac{10}{T}}$$

$$3^3 = 3^{\frac{10}{T}}$$

exp. only

$$3 = \frac{10}{T}$$

$$T = \frac{10}{3} = 3\frac{1}{3} \text{ hr}$$

$$T = 30 \text{ hr } 20 \text{ min}$$

Chapter 8 : Logarithmic Functions

1. For the equation $y = 3\log_5(6(x+2)) - 4$, state:

a) Domain

$$\{x \mid x > -2, x \in \mathbb{R}\}$$

b) Range

$$\{y \mid y \in \mathbb{R}\}$$

c) Equation of the asymptote

$$x = -2$$

d) x -intercept (if it exists)

$$0 = 3\log_5(6(x+2)) - 4$$

$$\frac{4}{3} = \log_5(6(x+2))$$

$$5^{\frac{4}{3}} = 6(x+2)$$

$$5^{\frac{4}{3}} = 6x + 12$$

$$\frac{5^{\frac{4}{3}} - 12}{6} = x$$

$$x = -0.575$$

e) y -intercept (if it exists)

$$y = 3\log_5(6(0+2)) - 4$$

$$= 3\log_5 12 - 4$$

$$= 3 \cdot \frac{\log 12}{\log 5} - 4$$

$$y = 0.632$$

2. Simplify to a single log and then evaluate (if possible).

$$a) 2\log_2 12 - \left(\log_2 6 + \frac{1}{3}\log_2 27\right)$$

$$= \log_2 12^2 - \log_2 6 - \log_2 27^{\frac{1}{3}}$$

$$= \log_2 144 - \log_2 6 - \log_2 3$$

$$= \log_2 \left(\frac{144}{6 \cdot 3} \right)$$

$$= \log_2 (8)$$

$$= \log_2 2^3$$

$$= \boxed{3}$$

$$b) 2\log_5 4 + \log_5 3 - \log_5 11$$

$$= \log_5 \left(\frac{4^2 \cdot 3}{11} \right)$$

$$= \log_5 \left(\frac{48}{11} \right)$$

$$c) \log x - 3 \log y + \frac{2}{3} \log z$$

$$= \log \left(\frac{x \cdot z^{\frac{2}{3}}}{y^3} \right)$$

$$d) \log_2(x+2) + \log_4 x \quad \text{bases don't match}$$

$$= \log_2(x+2) + \frac{\log_2 x}{\log_2 4}$$

$$= \log_2(x+2) + \frac{\log_2 x}{2}$$

$$= \log_2(x+2) + \frac{1}{2} \log_2 x$$

$$= \log_2(x+2) + \log_2 x^{\frac{1}{2}}$$

$$= \log_2((x+2)(x^{\frac{1}{2}}))$$

$$= \log_2(x^{\frac{3}{2}} + 2x^{\frac{1}{2}})$$

3. Solve. Express your answer to the nearest hundredth, if necessary.

$$a) \log_7(2x-3) - \log_7(x+2) = 1$$

$$\log_7 \left(\frac{2x-3}{x+2} \right) = 1$$

$$7^1 = \frac{2x-3}{x+2}$$

$$7(x+2) = 2x-3$$

$$7x+14 = 2x-3$$

$$\text{rest: } 2x-3 > 0$$

$$x+2 > 0$$

$$x > \frac{3}{2}$$

$$x > -2$$

$$5x = -17$$

$$x = -\frac{17}{5} \quad \left. \begin{array}{l} \text{rest. not ok} \\ \rightarrow \text{no solution} \end{array} \right\}$$

$$b) \log_b(x+2) - \log_b 4 = \log_b 3x$$

$$\log_b \left(\frac{x+2}{4} \right) = \log_b 3x$$

$$\frac{x+2}{4} = 3x$$

$$x+2 = 12x$$

$$2 = 11x$$

$$\frac{2}{11} = x$$

rest. ok!

$$\text{rest: } x+2 > 0$$

$$3x > 0$$

$$x > -2$$

$$x > 0$$

$$c) 2\log_4(x+4) - \log_4(x+12) = 1$$

$$\log_4 \left(\frac{(x+4)^2}{(x+12)} \right) = 1$$

$$4^1 = \frac{(x+4)^2}{(x+12)}$$

$$4x+48 = x^2 + 8x + 16$$

$$0 = x^2 + 4x - 32$$

$$0 = (x+8)(x-4)$$

$$\text{rest: } x+4 > 0$$

$$x+12 > 0$$

$$x > -4$$

$$x > -12$$

$$0 = (x+8)(x-4)$$

$$\downarrow \quad \downarrow$$

$$x = -8$$

$$x = 4$$

does not
meet rest.

c) $\frac{2\ln(5x-2)}{x} = \frac{16}{2}$ rest. $x > \frac{2}{5}$

$$\ln(5x-2) = 8$$

$$e^8 = 5x-2$$

$$\frac{e^8 + 2}{5} = x$$

$$\{ 596.6 = x$$

4. Solve. Express your answer to the nearest hundredth, if necessary.

a) $9^{2x-1} = 71^{x+2}$

$$(2x-1)\log 9 = (x+2)\log 71$$

$$2x\log 9 - \log 9 = x\log 71 + 2\log 71$$

$$2x\log 9 - x\log 71 = 2\log 71 + \log 9$$

$$x(2\log 9 - \log 71) = 2\log 71 + \log 9$$

$$x = \frac{2\log 71 + \log 9}{2\log 9 - \log 71}$$

$$\{ x = 81.37$$

c) $e^{3x+1} = 2$

$$(3x+1)\ln e = \ln 2$$

$$3x+1 = \ln 2$$

$$x = \frac{\ln 2 - 1}{3}$$

$$\{ x = -0.10$$

b) $4(7^{x+2}) = 9^{2x-3}$

$$\log 4 + (x+2)\log 7 = (2x-3)\log 9$$

$$\log 4 + x\log 7 + 2\log 7 = 2x\log 9 - 3\log 9$$

$$\log 7 - 2x\log 9 = -3\log 9 - \log 4 - 2\log 7$$

$$\times (\log 7 - 2\log 9) = -3\log 9 - \log 4 - 2\log 7$$

$$x = \frac{-3\log 9 - \log 4 - 2\log 7}{\log 7 - 2\log 9}$$

$$\{ x = 4.85$$

Chapter 9 : Rational Functions



1. For each function, find the locations of any vertical asymptotes, points of discontinuity, and intercepts.

a) $y = \frac{x^2+4x}{x^2+9x+20}$

$$= \frac{x(x+4)}{(x+4)(x+5)} = \frac{x}{x+5}$$

pt. of disc. at $x = -4$

$$y = \frac{-4}{-4+5} = -4$$

$(-4, -4)$

vert. asymptote at $x = -5$

x-int.

$x = 0$

y-int.

$$y = \frac{0}{0+5} \quad y = 0$$

b) $y = \frac{2x^2-5x-3}{x^2-1}$

$$= \frac{(x-3)(2x+1)}{(x+1)(x-1)}$$

no point of disc.

vert. asymptotes at $x = -1, x = 1$

x-int.

$x = 3$

$x = -\frac{1}{2}$

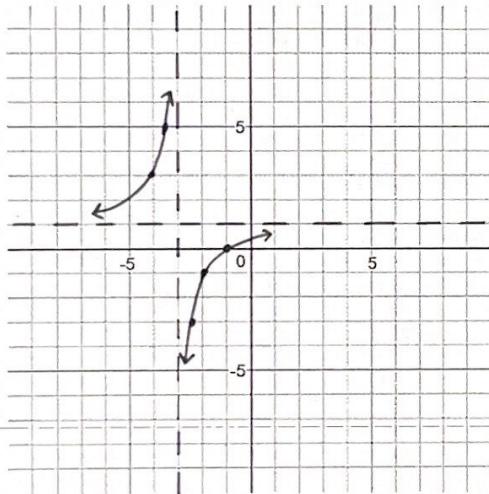
y-int.

$$y = \frac{(0-3)(2 \cdot 0 + 1)}{(0+1)(0-1)} = \frac{(-3)(1)}{(1)(-1)} \quad y = 3$$

2. Graph the following functions using transformations and show at least 6 points. Label/identify any asymptotes.



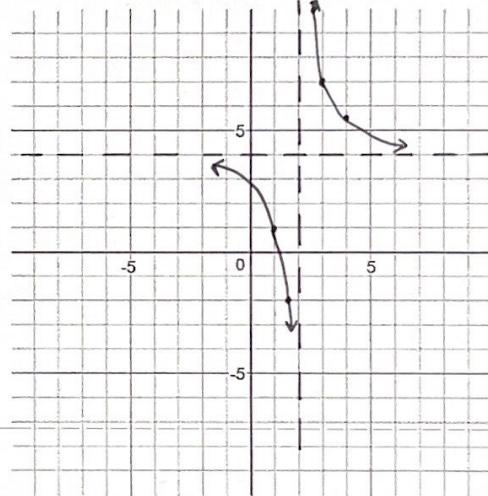
a) $y = \frac{-2}{x+3} + 1$



-3	x	y	-2	+1
-4	-1	-2	2	3
-3.5	-1/2	-2	4	5
-3	0	undefined		
-2.5	1/2	2	-4	-3
-2	1	+	-2	-1
-1	2	1/2	-1	0

b) $y = \frac{4x-5}{x-2}$

$$x-2 \sqrt{4x-5} - \frac{4x-8}{3} \quad y = \frac{3}{x-2} + 4$$



+2	x	y	.3	+4
1	1	-1	-3	1
1.5	1/2	-2	-6	-2
2	0	undefined		
2.5	1/2	2	6	10
3	1	+	3	7
4	2	1/2	15	5.5

Chapter 10 : Composite Functions

1. If $f(x) = \sqrt{x+2}$ and $g(x) = |2x|$; find $f \circ g(-7)$

$$f \circ g(-7) = f(g(-7))$$

$$g(-7) = |2(-7)| = |-14| = 14$$

$$f(14) = \sqrt{14+2} = \sqrt{16} = 4$$

$$\text{f } \circ \text{g } (-7) = 4$$

2. If $f(x) = x^2 + 7$ and $g(x) = 2x - 1$; find $f(g(x))$

$$\begin{aligned} f(g(x)) &= (2x-1)^2 + 7 \\ &= (2x-1)(2x-1) + 7 \\ &= 4x^2 - 2x - 2x + 1 + 7 \end{aligned}$$

$$f(g(x)) = 4x^2 - 4x + 8$$

Chapter 12 : Geometric Sequences and Series

1. How many terms are in the geometric sequence 2, 6, 18, ..., 486

$$n = ?$$

$$t_1 = 2$$

$$r = 3$$

$$t_n = 486$$

$$t_n = t_1 \cdot r^{n-1}$$

$$\frac{486}{2} = \frac{2 \cdot 3^{n-1}}{2}$$

$$243 = 3^{n-1}$$

$$3^5 = 3^{n-1}$$

$$\text{exp only : } 5 = n - 1$$

$$6 = n$$

2. The sum of an infinite series is 63 and the first term is 21. Find the common ratio.

$$S_{\infty} = 63$$

$$t_1 = 21$$

$$r = ?$$

$$S_{\infty} = \frac{t_1}{1-r}$$

$$63 = \frac{21}{1-r}$$

$$63(1-r) = 21$$

$$63 - 63r = 21$$

$$-63r = -42$$

$$r = \frac{2}{3}$$

3. Find the sum of the first 12 terms of the following geometric series: $12 + 4 + \frac{4}{3} + \dots$

$$n = 12$$

$$t_1 = 12$$

$$r = \frac{1}{3}$$

$$S_{12} = ?$$

$$S_n = \frac{t_1(r^n - 1)}{r-1}$$

$$S_{12} = \frac{12((1/3)^{12} - 1)}{(1/3) - 1}$$

$$= \frac{-11.999977}{(-2/3)}$$

$$S_{12} = 17.999966 \dots$$

$$S_{12} \approx 18$$