2.1 Radical Functions and Transformations

Radical Function: A function that involves a radical. The variable is in the radicand. ex: $y = \sqrt{\boxed{3x+2}}$ radicand

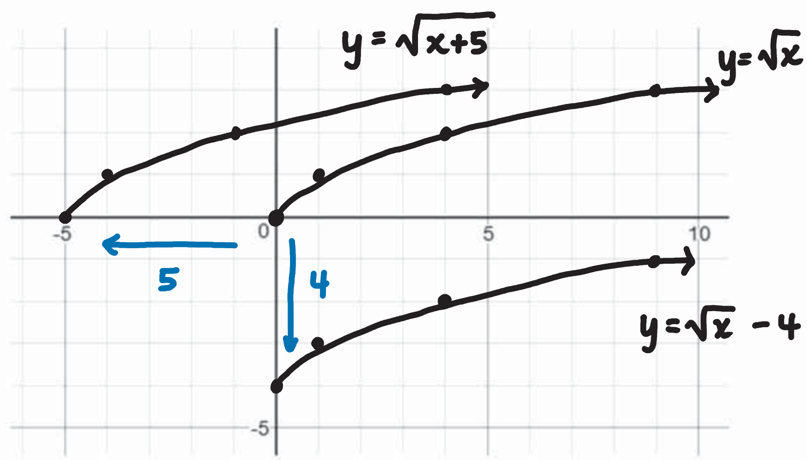
Example 1: Use a table of values to sketch the graph of each function. State the domain and range for each graph.

a) $y = \sqrt{x}$ "average joe" radical function

x	y
0	0
1	1
4	2
9	3

Domain: $\{x \mid x \geq 0 \quad x \in \mathbb{R}\}$

Range: $\{y \mid y \geq 0 \quad y \in \mathbb{R}\}$



b) $y = \sqrt{x+5}$ horizontal translation left 5 units

x	y
-5	0
-4	1
-1	2
4	3

D: $\{x \mid x \geq -5 \quad x \in \mathbb{R}\}$

R: $\{y \mid y \geq 0 \quad y \in \mathbb{R}\}$

c) $y = \sqrt{x} - 4$ vertical translation down 4 units

x	y
0	-4
1	-3
4	-2
9	-1

D: $\{x \geq 0 \quad x \in \mathbb{R}\}$

R: $\{y \geq -4 \quad y \in \mathbb{R}\}$

Graphing Radical Functions using Transformations:

$$y = a\sqrt{b(x-h)} + k$$

a : vertical stretch factor of $|a|$
If $a < 0 \rightarrow$ reflection over x-axis

h: horizontal translation

b : horizontal stretch factor of $\left|\frac{1}{b}\right|$
If $b < 0 \rightarrow$ reflection over y-axis

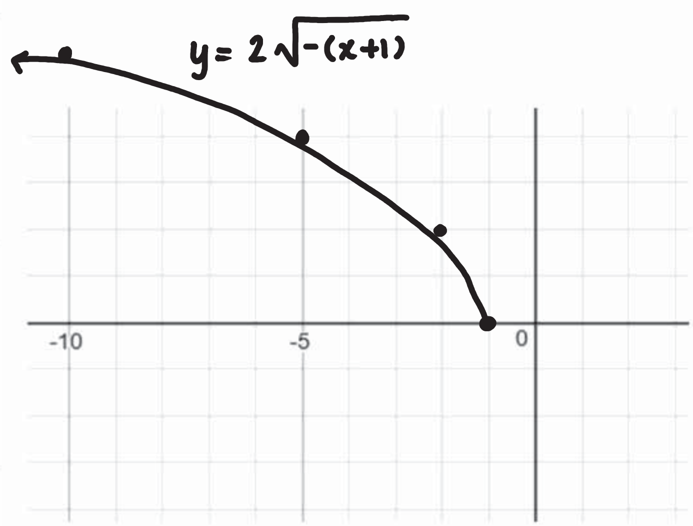
k: vertical translation

Example 2: Sketch the graph of the function $y = 2\sqrt{-(x+1)}$ by mapping individual points.

"avg joe" values

Transformation	Mapping
$a = 2$ vert. stretch factor of 2	$(0,0) \rightarrow (0,0)$ $(1,1) \rightarrow (1,2)$ $(4,2) \rightarrow (4,4)$ $(9,3) \rightarrow (9,6)$
$b = -1$ reflexion over y-axis	$(0,0) \rightarrow (0,0)$ $(1,2) \rightarrow (-1,2)$ $(4,4) \rightarrow (-4,4)$ $(9,6) \rightarrow (-9,6)$
$h = -1$ horizontal translation left 1 unit	$(0,0) \rightarrow (-1,0)$ $(-1,2) \rightarrow (-2,2)$ $(-4,4) \rightarrow (-5,4)$ $(-9,6) \rightarrow (-10,6)$

graph these values



State the domain and range for the function:

$D : \{x \mid x \le -1 \quad x \in \mathbb{R}\}$

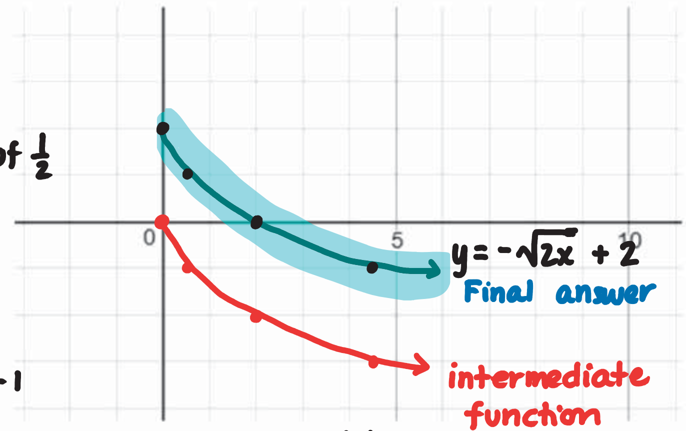
$R : \{y \mid y \ge 0 \quad y \in \mathbb{R}\}$

Example 3: Sketch the graph of the function $y - 2 = -\sqrt{2x}$ by transforming the graph directly. State the transformations.

rewrite

$\hookrightarrow y = -\sqrt{2x} + 2$

- $a = -1$ reflection over x-axis
- $b = 2$ horiz. stretch by factor of $\frac{1}{2}$
- $k = 2$ vert. translation up 2



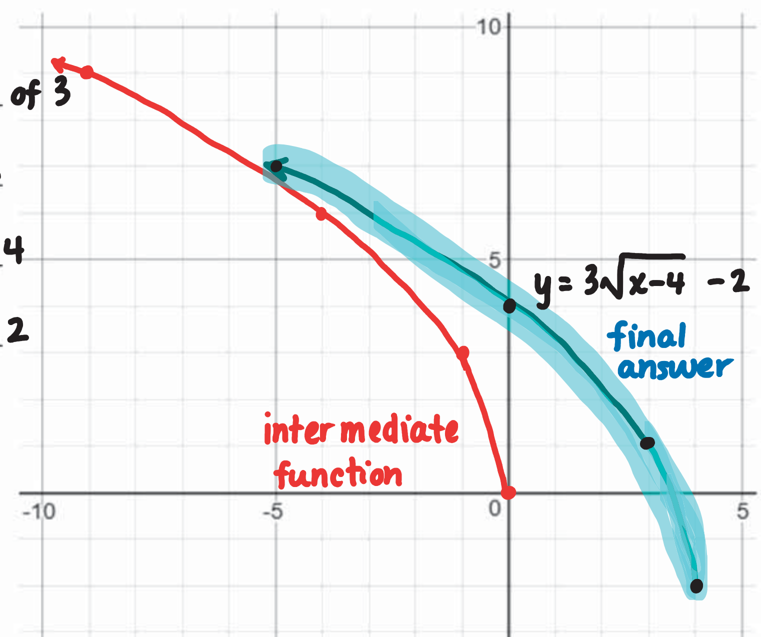
original ("avg. joe)

	x	y	
0	~~0~~	~~0~~	0
½	~~1~~	~~1~~	−1
2	~~4~~	~~2~~	−2
9/2	~~9~~	~~3~~	−3

- mult. "y" by −1
- divide "x" by 2 (or mult. "x" by $\frac{1}{2}$)
- trans. "y" up 2

Example 4: Sketch the graph of the function $y = 3\sqrt{-(x-4)} - 2$ State the transformations.

- $a = 3$ vert. stretch by factor of 3
- $b = -1$ reflection over y-axis
- $h = 4$ horiz. translation right 4
- $k = -2$ vert. translation down 2

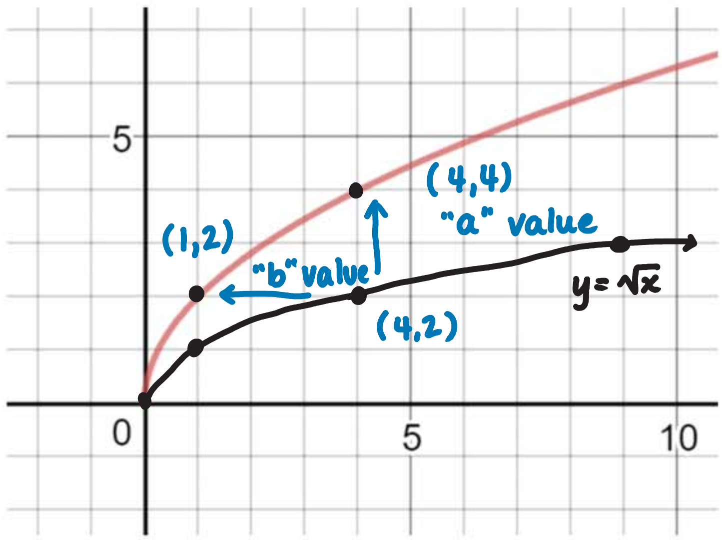


original

	x	y	
0	~~0~~	~~0~~	0
−1	~~1~~	~~1~~	3
−4	~~4~~	~~2~~	6
−9	~~9~~	~~3~~	9

- mult. "y" by 3
- mult. "x" by −1
- trans. "x" right 4
- trans. "y" down 2

Example 5: State the equation for the given graph.



- looks like a radical function
- compare to "avg. joe" $y = \sqrt{x}$
- $h = 0$ and $k = 0$ (no translations)

View as a vertical stretch

$y = a\sqrt{x}$

$(4,2) \longrightarrow (4,4)$

use point on the new function to find "a".

$(1,2)$ or $(4,4)$

$y = a\sqrt{x}$

$2 = a\sqrt{1}$ $\quad$ $4 = a\sqrt{4}$

$2 = a(1)$ $\quad$ $\frac{4}{2} = \frac{a(2)}{2}$

$2 = a$ $\quad$ $2 = a$

$y = 2\sqrt{x}$

View as a horizontal stretch

$y = \sqrt{bx}$

$(4,2) \longrightarrow (1,2)$

use point on the new function to find "b"

$(1,2)$ or $(4,4)$

$y = \sqrt{bx}$

$2 = \sqrt{b(1)}$ $\quad$ $4 = \sqrt{b(4)}$

$(2)^2 = (\sqrt{b})^2$ $\quad$ $(4)^2 = (\sqrt{4b})^2$

$4 = b$ $\quad$ $\frac{16}{4} = \frac{4b}{4}$

$\quad$ $4 = b$

$y = \sqrt{4x}$

Practice: p.72 #1cd, 2 – 6, 8, 10, 11ab, 16