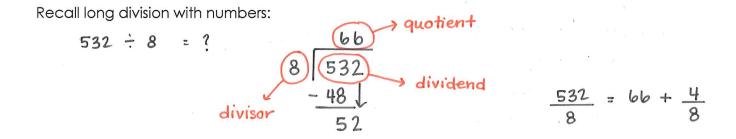
3.2 The Remainder Theorem



A. Long Division with Polynomials

Example 1: Divide $2x^{(3)} - 4x^2 + 3x - 6$ by x + 2

$$\begin{array}{c|c}
2x^{2} - 8x + 19 \\
\hline
(x) + 2 \overline{)2x^{3} - 4x^{2} + 3x - 6} \\
- (2x^{3} + 4x^{2}) \\
\hline
- 8x^{2} + 3x \\
- (-8x^{2} - 16x) \\
\hline
- 19x - 6 \\
- (19x + 38) \\
- 44
\end{array}$$
Dividend must be written in descending order

Polynomial,
$$P(x) = 2x^3 - 4x^2 + 3x - 6$$

Quotient, $Q(x) = 2x^2 - 8x + 19$
Remainder, $R = -44$

Division Statement: When P(x) is divided by x - a

$$\frac{P(x)}{x-a} = Q(x) + \frac{R}{x-a} \qquad \text{or} \qquad P(x) = Q(x)(x-a) + R$$

$$\frac{P(x)}{2x^3 - 4x^2 + 3x - b} = 2x^2 - 8x + 19 + (-44)$$

$$\frac{2x^3 - 4x^2 + 3x - b}{x + 2} = 2x^2 - 8x + 19 + (-44)$$

$$\frac{2x^3 - 4x^2 + 3x - b}{x + 2} = (2x^3 - 4x^2 + 3x - b) = (2x^3 - 8x + 19)(x + 2) - 44$$

$$\frac{2x^3 - 8x + 19}{x + 2} = (2x^3 - 8x + 19)(x + 2) - 44$$

Mrs. Donnelly

Example 2: Divide $\frac{x^3-6-6x}{x-2}$ } Rewrite in descending order; hold a position for the missing term $(0x^2)$.

$$\begin{array}{c|c}
x^{2} + 2x - 2 \\
x^{3} + 0x^{2} - bx - 6 \\
-(x^{3} - 2x^{2}) \downarrow \\
2x^{2} - bx \\
-(2x^{2} - 4x) \downarrow \\
-2x - 6 \\
-(-2x + 4) \\
-10
\end{array}$$

$$\frac{(x^{3}-bx-b)}{x-2} = x^{2}+2x-2+\frac{(-10)}{x-2}$$
or
$$x^{3}-bx-b = (x^{2}+2x-2)(x-2)+(-10)$$

$$P(x) = x^3 - bx - b$$

$$Q(x) = x^2 + 2x - 2$$

$$R = -10$$

B. Synthetic Division with Polynomials

Synthetic division is an alternative method of polynomial division.

Example 3:
$$(x^3 + 2x^2 - 7x - 2) \div (x - 1)$$

$$Q(x) = x^2 + 3x - 4$$

 $R = -6$

- 1) write each coeff. of the dividend, P(x)
- 2 to the left, write value of "-1" from the
 - 3 bring down 1st coefficient
 - 4) multiply (-1) by I and write product in 2nd column
 - (5) subtract 2nd column (2-(-1))
 - 6 continue steps 4 \$5

$$\frac{p(x)}{x-1} = x^2 + 3x - 4 + \frac{-6}{x-1}$$

C. The Remainder Theorem

The Remainder Theorem states that when a polynomial P(x) is divided by (x - a), the remainder is P(a).

Remainder R = P(a) when dividing P(x) by (x - a).

Example 5: Find the remainder when $P(x) = x^3 - 8x^2 + x + 37$ is divided by (x - 2).

a = 2, so find P(2)

$$R = P(2) = (2)^{3} - 8(2)^{2} + (2) + 37$$

$$= 8 - 24 + 2 + 37$$

$$= 8 - 15$$

check using synthetic
$$-2$$
 | 1 -8 | 1 37 | $-$ | 1 -2 | 1 22 | 1 -6 -11 | 15 | Remainder !

Example 6: When $P(x) = x^3 - kx^2 + 17x + 6$ is divided by (x - 3), the remainder is 12. Find k.

$$a = 3 R = 12$$

$$P(a) = R$$

$$P(3) = 12$$

$$(3)^{3} - K(3)^{2} + 17(3) + b = 12$$

$$27 - 9K + 51 + b = 12$$

$$84 - 9K = 12$$

$$-84 - 84$$

$$- 9K = -72$$

$$-9 -9$$

$$K = 8$$

^{*} assessed on both division methods so practice both