3.2 The Remainder Theorem

Recall long division with numbers:

$532 \div 8 = ?$

quotient

$$\begin{array}{r|l} & 66 \\ \hline 8 & 532 \\ & -48\downarrow \\ & \quad 52 \\ & \,-48 \\ & \quad\ \ 4 \end{array}$$

divisor

dividend

remainder

$$\frac{532}{8} = 66 + \frac{4}{8}$$

A. Long Division with Polynomials

Example 1: Divide $2x^3 - 4x^2 + 3x - 6$ by $x + 2$

$$\begin{array}{r|l} & 2x^2 - 8x + 19 \\ \hline x+2 & 2x^3 - 4x^2 + 3x - 6 \\ & -(2x^3 + 4x^2) \downarrow \\ & \qquad -8x^2 + 3x \\ & \quad -(-8x^2 - 16x) \downarrow \\ & \qquad\qquad 19x - 6 \\ & \qquad\quad -(19x + 38) \\ & \qquad\qquad\quad -44 \end{array}$$

what do we need to multiply "x" by in order to make it the same as "$2x^3$" ?

} Dividend must be written in descending order

Polynomial, $P(x) = 2x^3 - 4x^2 + 3x - 6$

Quotient, $Q(x) = 2x^2 - 8x + 19$

Remainder, $R = -44$

Division Statement: When $P(x)$ is divided by $x - a$

$$\frac{P(x)}{x-a} = Q(x) + \frac{R}{x-a} \qquad \text{or} \qquad P(x) = Q(x)(x-a) + R$$

$$\underbrace{\frac{2x^3 - 4x^2 + 3x - 6}{x+2}}_{\frac{P(x)}{x-a}} = \underbrace{2x^2 - 8x + 19}_{Q(x)} + \frac{\overset{R}{(-44)}}{\underset{x-a}{x+2}}$$

$$\underbrace{2x^3 - 4x^2 + 3x - 6}_{P(x)} = \underbrace{(2x^2 - 8x + 19)}_{Q(x)}\underbrace{(x+2)}_{x-a} \underbrace{- 44}_{R}$$

Example 2: Divide $\frac{x^3-6-6x}{x-2}$ } Rewrite in descending order; hold a position for the missing term $(0x^2)$.

$$
\begin{array}{r}
x^2+2x-2 \\
x-2 \overline{\big)\, x^3+0x^2-6x-6} \\
-(x^3-2x^2) \downarrow \qquad \\
\hline
2x^2-6x \qquad \\
-(2x^2-4x) \downarrow \\
\hline
-2x-6 \\
-(-2x+4) \\
\hline
-10
\end{array}
$$

$$\frac{x^3-6x-6}{x-2} = x^2+2x-2+\frac{(-10)}{x-2}$$

or

$$x^3-6x-6 = (x^2+2x-2)(x-2)+(-10)$$

$P(x) = x^3-6x-6$

$Q(x) = x^2+2x-2$

$R = -10$

B. Synthetic Division with Polynomials

Synthetic division is an alternative method of polynomial division.

Example 3: $(\overset{P(x)}{x^3+2x^2-7x-2}) \div (x-1)$

$$
\begin{array}{r|rrrr}
-1 & 1 & 2 & -7 & -2 \\
\text{subtraction } \ominus & \downarrow & -1 & -3 & 4 \\
\hline
\text{multiplication } \otimes & \underbrace{1 \quad 3 \quad -4}_{\text{coefficients of the quotient } Q(x)} & & & \underbrace{-6}_{\text{Remainder } R}
\end{array}
$$

① write each coeff. of the dividend, $P(x)$

② to the left, write value of "-1" from the factor $x-1$

③ bring down 1st coefficient

④ multiply (-1) by 1 and write product in 2nd column

⑤ subtract 2nd column $(2-(-1))$

⑥ continue steps 4 & 5

$Q(x) = x^2+3x-4$

$R = -6$

$$\frac{P(x)}{x-1} = x^2+3x-4+\frac{-6}{x-1}$$

Example 4: $(x^3 - 5x^2 + 10x - 15\ ) \div (x - 3)$

$$\begin{array}{c|cccc} -3 & 1 & -5 & 10 & -15 \\ - & \downarrow & -3 & 6 & -12 \\ \hline \times & \underbrace{1 \quad -2 \quad 4}_{Q(x)} & & & \underbrace{-3}_{R} \end{array}$$

$Q(x) = x^2 - 2x + 4$

$R = -3$

$$\frac{x^3 - 5x^2 + 10x - 15}{x - 3} = x^2 - 2x + 4 + \frac{(-3)}{x - 3}$$

C. The Remainder Theorem

The Remainder Theorem states that when a polynomial $P(x)$ is divided by $(x - a)$, the remainder is $P(a)$.

Remainder $R = P(a)$ when dividing $P(x)$ by $(x - a)$.

Example 5: Find the remainder when $P(x) = x^3 - 8x^2 + x + 37$ is divided by $(x - 2)$.

$a = 2$, so find $P(2)$

$$R = P(2) = (2)^3 - 8(2)^2 + (2) + 37$$

$$= 8 - \underset{32}{\cancel{24}} + 2 + 37$$

$$R = 15$$

check using synthetic division :

$$\begin{array}{c|cccc} -2 & 1 & -8 & 1 & 37 \\ - & \downarrow & -2 & 12 & 22 \\ \hline \times & 1 & -6 & -11 & \underbrace{15}_{\text{Remainder !}} \end{array}$$

Example 6: When $P(x) = x^3 - \boldsymbol{k}x^2 + 17x + 6$ is divided by $(x - 3)$, the remainder is 12. Find $\boldsymbol{k}$.

$a = 3 \qquad R = 12$

$P(a) = R$

$P(3) = 12$

$(3)^3 - k(3)^2 + 17(3) + 6 = 12$

$27 - 9k + 51 + 6 = 12$

$84 - 9k = 12$

$-84 \qquad -84$

$\frac{-9k}{-9} = \frac{-72}{-9}$

$k = 8$

★ assessed on both division methods so practice both.

Practice: p.124 #3cd, 4def, 5ef, 6ace, 7ab, 8cd, 9, 10, 15