3.4 Equations and Graphs of Polynomial Functions: Part 1

Find the zeros of the function

$f(x) = \frac{1}{2}(x-1)(x+2)(x-3)$

set $f(x) = 0$

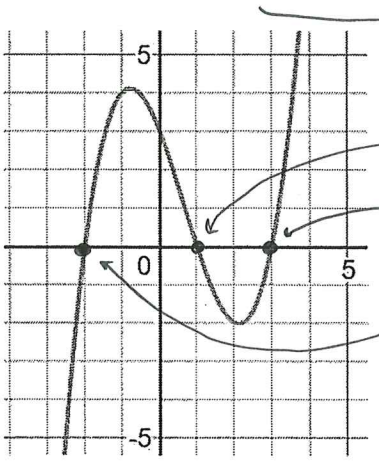
$0 = \frac{1}{2}(x-1)(x+2)(x-3)$

$x-1=0$ → $x=1$

$x+2=0$ → $x=-2$

$x-3=0$ → $x=3$

Find the x-intercepts of the function



where graph crosses x-axis

$x = 1$

$x = 3$

$x = -2$

The zeroes of a polynomial function are the x- intercepts of the graph of the function.

They are also known as roots

Multiplicity of a zero/root : how many times a particular number is a zero for a given polynomial.

$f(x) = (x-1)^2(x+4)$

Find the "zeroes"

$0 = (x-1)^2(x+4)$

$0 = (x-1)(x-1)(x+4)$

$x-1=0$

$x=1$

2 zeroes of $x=1$

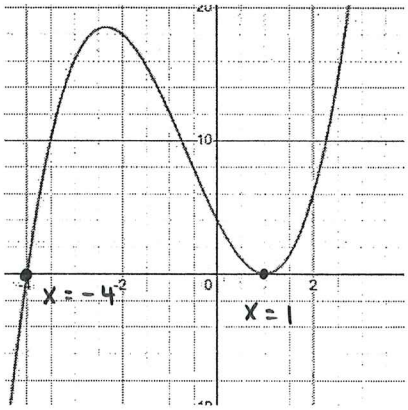
▸ Multiplicity of 2

$x+4=0$

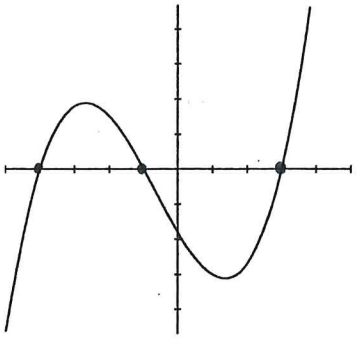
$x=-4$

1 zero of $x=-4$

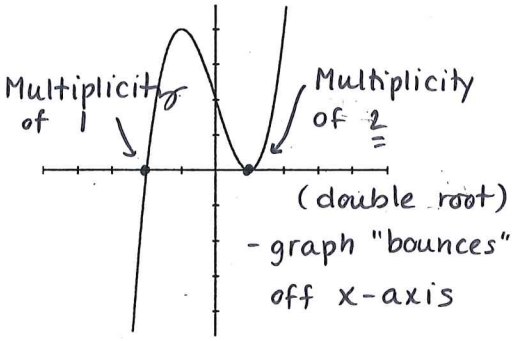
▸ Multiplicity of 1

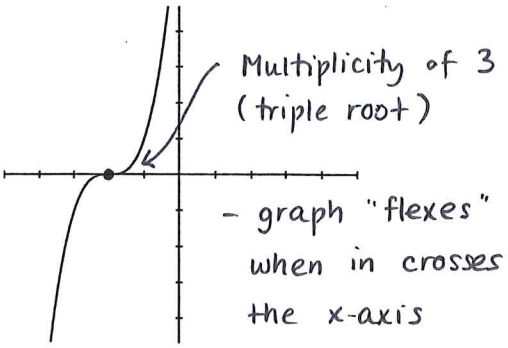


To determine the multiplicity of a zero/root from a graph, consider the following:



Multiplicity of 1
for each root/zero

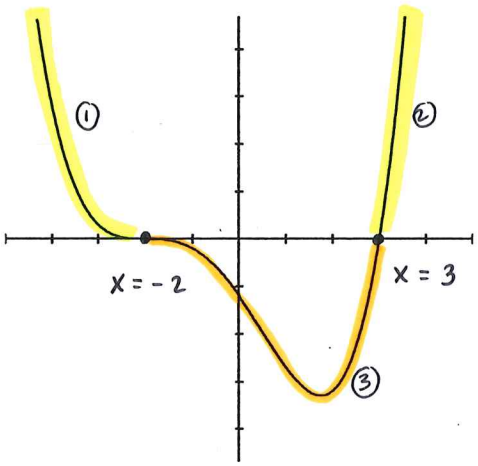




- Zeroes of **ODD** multiplicity change sign at the zero. (graph crosses x-axis)
- Zeroes of **EVEN** multiplicity do not change sign at the zero. (graph doesn't cross x-axis, it bounces off)

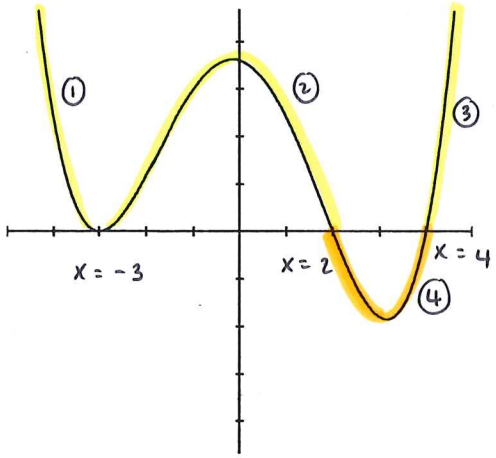
Example 1: For each graph, state the x-intercepts, the intervals where the function is positive and negative, whether the zeroes are of multiplicity 1,2, or 3.

a)



x – intercepts	$(-2, 0)$ and $(3, 0)$
Multiplicity	$x = -2$ mult. of 3 ; $x = 3$ mult of 1
Positive interval	(above x-axis) $x < -2$ and $x > 3$ ① ②
Negative interval	(below x-axis) $-2 < x < 3$ ③

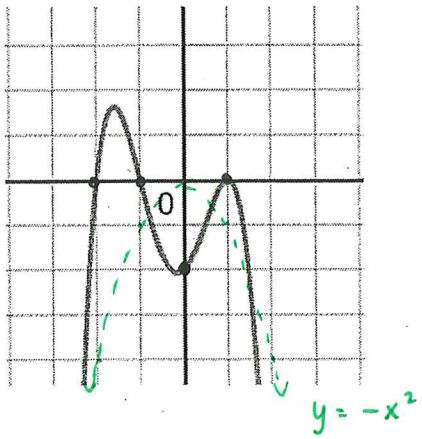
b)



x – intercepts	$(-3, 0)$, $(2, 0)$, $(4, 0)$
Multiplicity	$x = -3$ mult. of 2 ; $x = 2$ and $x = 4$ mult. of 1 each
Positive interval	① $x < -3$ ③ $x > 4$ ② $-3 < x < 2$
Negative interval	④ $2 < x < 4$

Example 2: For the following polynomial functions determine: the sign of the leading coefficient, the x-intercepts, multiplicity of the zeros, and an additional point. Use the information to find the equation of the polynomial function.

a)



$y = a(x+2)(x+1)(x-1)^2$

$-2 = a(0+2)(0+1)(0-1)^2$

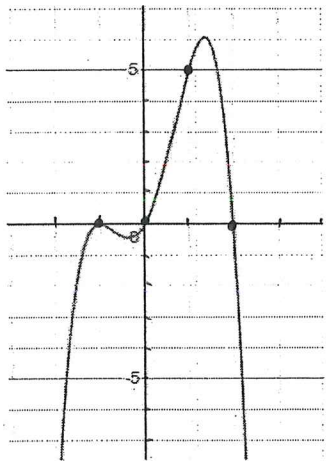
$-2 = 2a$

$-1 = a$

$y = -(x+2)(x+1)(x-1)$

Sign of Leading Coefficient	negative
x – intercepts	(-2,0) (1,0) (-1,0)
Multiplicity	(-2,0) (-1,0) mult. of 1; (1,0) mult. of 2
Additional Point	(0,-2)

b)



$y = a(x+1)^2(x)(x-2)$

$5 = a(1+1)^2(1)(1-2)$

$5 = a(2)^2(1)(-1)$

$5 = -4a$

$-\frac{5}{4} = a$

$y = -\frac{5}{4}(x+1)^2(x)(x-2)$

Sign of Leading Coefficient	negative
x – intercepts	(-1,0) (2,0) (0,0)
Multiplicity	(-1,0) (0,0) mult. of 1; (2,0) mult. of 2
Additional Point	(1,5)

c) A degree 4 polynomial function has zeroes of -4, 1 (both multiplicity 1) and -2 (multiplicity 2). The constant term of the function is -3. → y-intercept (0,-3)

$y = a(x+4)(x-1)(x+2)^2$

$-3 = a(0+4)(0-1)(0+2)^2$

$-3 = -16a$

$a = \frac{3}{16}$

$y = \frac{3}{16}(x+4)(x-1)(x+2)^2$

Sign of Leading Coefficient	unsure
x – intercepts	(-4,0) (-2,0) (1,0)
Multiplicity	(-4,0) (1,0) mult. of 1; (-2,0) mult. of 2
Additional Point	(0,-3)

Practice: p.147 # 3, 4, 14 and worksheet