

3.4 Equations and Graphs of Polynomial Functions: Part 2

Example 1: For each function: determine the x -intercepts, the degree and end behavior of the graph, the zeroes and their multiplicity, the y -intercept of the graph, intervals where the function is positive and negative. Sketch the function.

a) $f(x) = x(x+3)(x-2)$

x -intercepts

$$0 = x(x+3)(x-2)$$

$\swarrow \quad \downarrow \quad \searrow$

$x=0 \quad x=-3 \quad x=2$

$(0,0) \quad (-3,0) \quad (2,0)$

degree and end behavior

degree = 3 (odd)

leading coefficient = positive

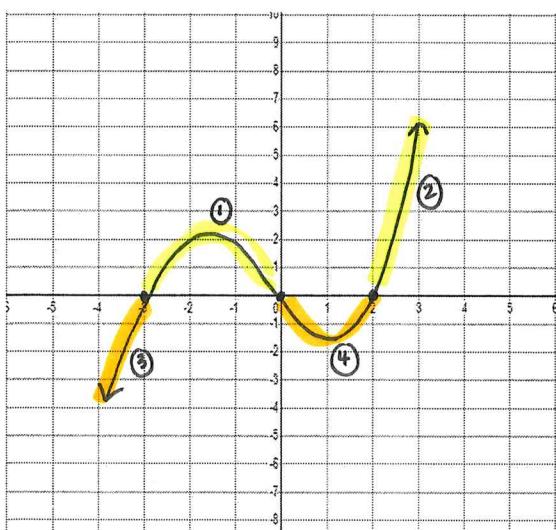
end behavior: down in quad III
up in quad I

zeroes and multiplicity

$(0,0) \quad (-3,0) \quad (2,0)$

all have multiplicity of 1

Graph



y -intercept \rightarrow constant term

or set $x = 0$

$$f(x) = 0(0+3)(0-2)$$

$$f(x) = 0$$

$$y\text{-int} : (0,0)$$

intervals of positive and negative

positive: $-3 < x < 0$, $x > 2$

negative: $x < -3$, $0 < x < 2$

b) $g(x) = x^4 + 2x^3 - 3x^2 - 8x - 4$

x -intercepts

factors of $-4 : \{ \pm 1, \pm 2, \pm 4 \}$

try $x = -1$ $g(-1) = (-1)^4 + 2(-1)^3 - 3(-1)^2 - 8(-1) - 4$

$g(-1) = 0$ so $x+1$ is a factor

1	1	2	-3	-8	-4
-	↓	1	1	-4	-4
x	1	1	-4	-4	0

$g(x) = (x+1)(x^3 + x^2 - 4x - 4)$ Now factor this

try $x = -1$ $(-1)^3 + (-1)^2 - 4(-1) - 4 = 0$
(again) so $x+1$ is another factor

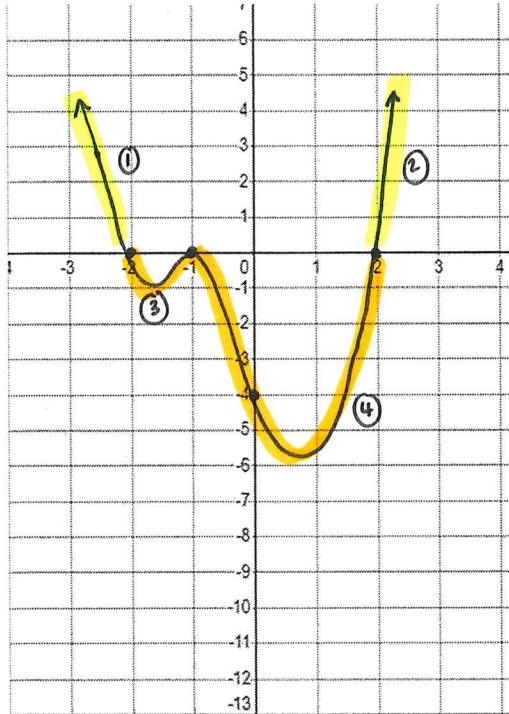
1	1	-4	-4
-	↓	1	0
x	1	0	-4

zeroes and multiplicity

$x = -1$ mult. of 2

$x = 2$ } mult. of 1 each
 $x = -2$ }

Graph



degree and end behavior

degree = 4

leading coeff = positive

end behavior: up in quad II

up in quad I

$\rightarrow g(x) = (x+1)(x+1)(x^2 - 4)$

$\rightarrow g(x) = (x+1)^2(x-2)(x+2)$

x -intercepts are:

$0 = (x+1)^2(x-2)(x+2)$

$\downarrow \quad \downarrow \quad \downarrow$
 $x = -1 \quad x = 2 \quad x = -2$

y -intercept

constant term

$y = -4$ or $(0, -4)$

intervals of positive and negative

positive: $x < -2$, $x > 2$

negative: $-2 < x < -1$, $-1 < x < 2$

(3) (4)

c) $f(x) = -2x^3 + 6x - 4$

x -intercepts

factors of -4 $\{\pm 1, \pm 2, \pm 4\}$

try $x = 1$ $-2(1)^3 + 6(1) - 4 = 0$
so $x-1$ is a factor

$$\begin{array}{r|rrrr} -1 & -2 & 0 & 6 & -4 \\ \hline & \downarrow & 2 & 2 & -4 \\ x & -2 & -2 & 4 & 0 \end{array}$$

$$f(x) = (x-1)(-2x^2 - 2x + 4)$$

Factor further

$$f(x) = -2(x-1)(x^2 + x - 2)$$

$$f(x) = -2(x-1)(x+2)(x-1)$$

$$f(x) = -2(x-1)^2(x+2)$$

zeroes and multiplicity

$x = 1$ mult. of 2

$x = -2$ mult. of 1

degree and end behavior

degree = 3

leading coeff. = negative

end behavior : up in quad II
down in quad IV

x -int. are :

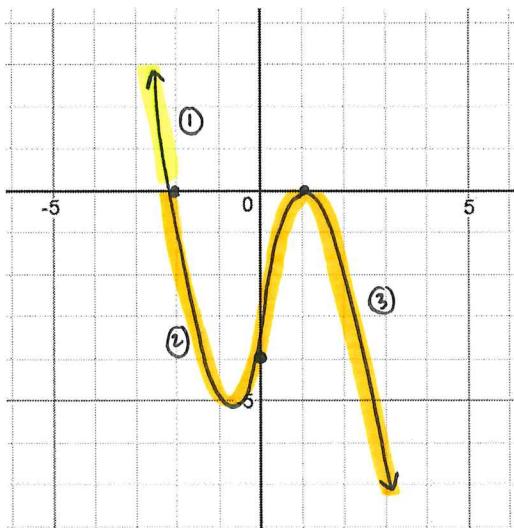
$$0 = -2(x-1)^2(x+2)$$

$$\begin{matrix} \downarrow & \downarrow \\ x = 1 & x = -2 \end{matrix}$$

y -intercept

$$y = -4 \quad \text{or} \quad (0, -4)$$

Graph



Intervals of positive and negative

positive : $x < -2$

negative : $-2 < x < 1$, $x > 1$