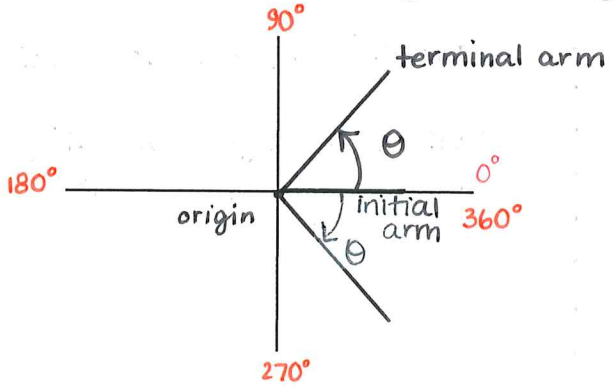
4.1 Angles and Angle Measure



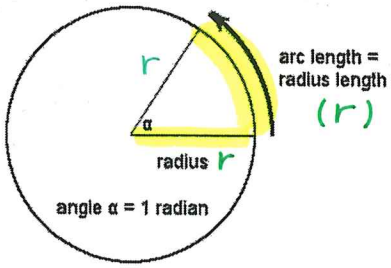
$\theta > 0$ (positive) measured counter-clockwise

$\theta < 0$ (negative) measured clockwise

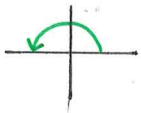
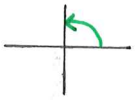
Angles in standard position have their center at the origin (0,0) and the initial arm on the positive x-axis.

Angles are measured in degrees or radians (new!).

An angle that has a measure of 1 radian is an angle in which the length of the radius = length of the arc of the angle.



In other words, 1 radian is the angle made when we take the radius and wrap it around the circle.

		Degrees	Radian Measure (no units)
Full rotation of a circle		360°	2π
Half rotation of a circle		180°	π
¼ rotation of a circle		90°	$\frac{\pi}{2}$

Note: Angles without units are considered radians.

Conversion factor : degrees and radians

$$\frac{\text{radians}}{\pi} = \frac{\text{degrees}}{360^\circ}$$

convert radians to degrees

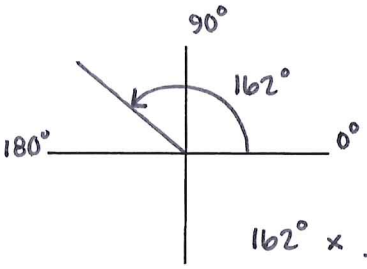
$$\underset{\text{(want)}}{\text{degrees}} = \underset{\text{(have)}}{\text{radians}} \times \frac{180^\circ}{\pi}$$

convert degrees to radians

$$\underset{\text{(want)}}{\text{radians}} = \underset{\text{(have)}}{\text{degrees}} \times \frac{\pi}{180^\circ}$$

Example 1: Convert each angle into degrees or radians. Draw each angle in standard position.

a) 162°



$$162^\circ \times \frac{\pi}{180^\circ}$$

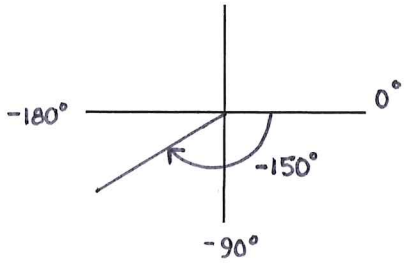
$$= \frac{162^\circ \pi}{180^\circ} \quad (\div 2)$$

$$= \frac{81\pi}{90} \quad (\div 9)$$

$$= \frac{9\pi}{10}$$

answer in fraction form

b) −150°



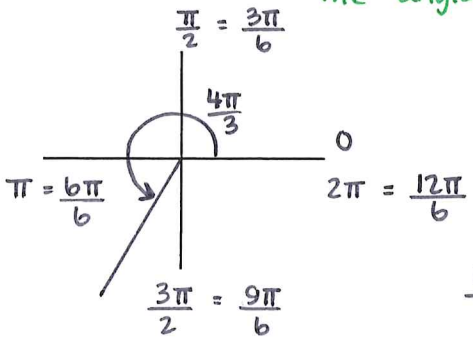
$$-150^\circ \times \frac{\pi}{180^\circ}$$

$$= \frac{-150^\circ \pi}{180^\circ} \quad (\div 10)$$

$$= \frac{-15^\circ \pi}{18} \quad (\div 3)$$

$$= \frac{-5\pi}{6}$$

c) $\frac{4\pi}{3} = \frac{8\pi}{6}$ } using a denominator of 6 helps to draw the angle

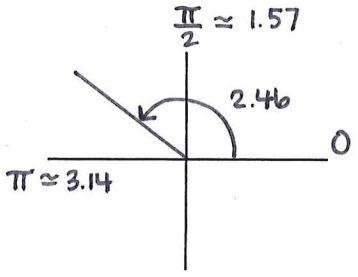


$$\frac{4\pi}{3} \times \frac{180^\circ}{\pi}$$

$$= \frac{720^\circ}{3}$$

$$= 240^\circ$$

d) 2.46 } no units, so this is in radians



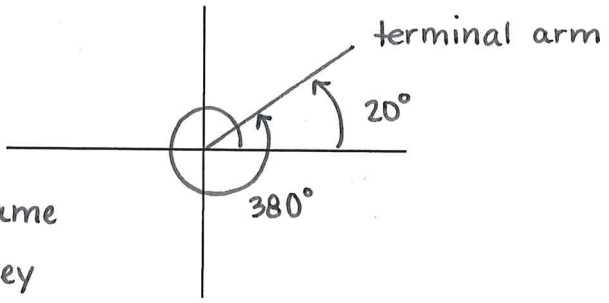
$$2.46 \times \frac{180^\circ}{\pi}$$

$$= \frac{442.8^\circ}{\pi}$$

$$= 140.9^\circ$$

Coterminal Angles: Angles that have the same terminal arm when in standard position.

Draw an angle of 20° and one of 380°.



20° & 380° share same terminal arm so they are coterminal.

We can find coterminal angles by adding or subtracing 360° (degrees)

or adding or subtracting 2π (radians)

The general form to express the coterminal angles is $\theta \pm 360°n$, $n \in N$

$\theta \pm 2\pi n$, $n \in N$

n represents any natural number (represents # of full rotations)

Example 2: Find two coterminal angles for each given angle. Express your answer in general form.

a) 150°

θ_1 (one full rotation, $n = 1$)

$\theta_1 = 150° + 360°n$

$= 150° + 360°(1)$

$\theta_1 = 510°$

θ_2 (one full negative rotation, $n=1$)

$\theta_2 = 150° - 360°n$

$= 150° - 360°(1)$

$\theta_2 = -210°$

general form: $\theta = 150° \pm 360°n, n \in N$

b) $\frac{\pi}{3}$

θ_1, $n = 1$ (pos. rot.)

$\theta_1 = \theta + 2\pi n$

$= \frac{\pi}{3} + 2\pi(1) \cdot \frac{3}{3}$

$= \frac{\pi}{3} + \frac{6\pi}{3}$

$\theta_1 = \frac{7\pi}{3}$

θ_2, $n=1$ (neg. rot.)

$\theta_2 = \theta - 2\pi n$

$= \frac{\pi}{3} - 2\pi(1) \cdot \frac{3}{3}$

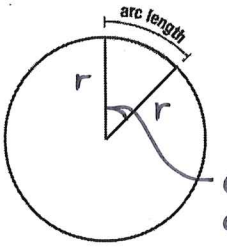
$= \frac{\pi}{3} - \frac{6\pi}{3}$

$\theta_2 = -\frac{5\pi}{3}$

general form: $\theta = \frac{\pi}{3} \pm 2\pi n, n \in N$

Arc Length of a Circle

Arc Length is the length of the arc that subtends (contains) the central angle.



using proportions :

$$\frac{\text{arc length}}{\text{circumference}} = \frac{\text{central angle}}{\text{full rotation}}$$

remember : $C = 2\pi r$ (r: radius, C: circumference)

Arc length in degrees

let x = arc length

$$\frac{x}{2\pi r} = \frac{\theta}{360^\circ}$$

$$x = \frac{2\pi r\theta}{360^\circ} \rightarrow x = \frac{\pi r\theta}{180^\circ}$$

Arc length in radians

$$\frac{x}{2\pi r} = \frac{\theta}{2\pi}$$

$$x = \frac{2\pi r\theta}{2\pi} \rightarrow x = r\theta$$

or $\frac{r\theta \cdot \pi}{180^\circ}$ conversion factor

Example 3: Find the arc length of the sector that is formed if

a) The central angle is 120° and the radius of the circle is 15 cm.

$\theta = 120^\circ$

$r = 15$ cm

$x = ?$

$$x = \frac{\pi r\theta}{180^\circ} = \frac{\pi(15)(120^\circ)}{180^\circ} = \frac{1800^\circ\pi}{180^\circ}$$

$x = 10\pi$ cm } exact value

$x = 31.4$ cm } approx. value

b) The central angle is $\frac{3\pi}{4}$ and the radius of the circle is 8 units.

$\theta = \frac{3\pi}{4}$

$r = 8$ units

$x = ?$

$$x = \theta r = \frac{3\pi}{4} \cdot 8 = \frac{24\pi}{4}$$

$x = 6\pi$ units exact

$x = 18.8$ units approx.

Practice: p.175 #2 – 7ace, 8ac, 9, 11adg, 13, 14