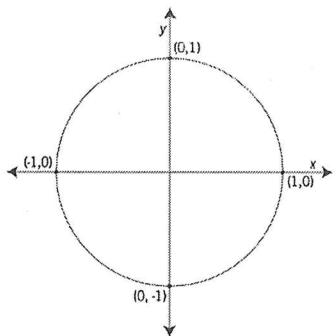


4.2 The Unit Circle

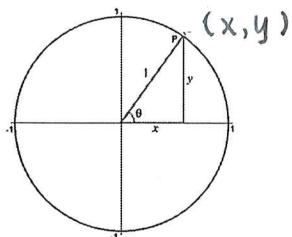


Unit circle:

A circle with a radius of 1 unit

and its center is at the origin

The equation of the unit circle can be found using the Pythagorean Theorem.



$$a^2 + b^2 = c^2$$

$$x^2 + y^2 = r^2$$

$$x^2 + y^2 = 1$$

Therefore, the equation of the unit circle is: $x^2 + y^2 = 1$

In general, the equation of any circle is: $x^2 + y^2 = r^2$

Example 1: Determine the equation of a circle with radius of 5 and the center at the origin.

$$r = 5$$

$$x^2 + y^2 = r^2$$

$$x^2 + y^2 = (5)^2$$

$$x^2 + y^2 = 25$$

Example 2: Is the following point on the unit circle? $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

Verify if $x^2 + y^2 = 1$
for this point

$$\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 ?= 1$$

$$\frac{1}{4} + \frac{3}{4} ?= 1$$

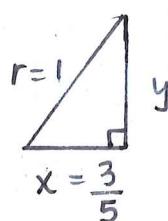
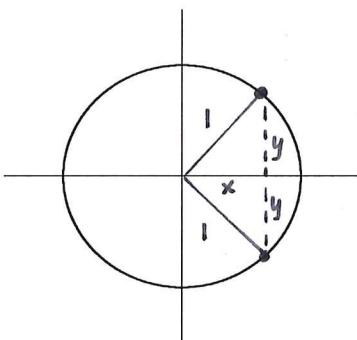
$$\frac{4}{4} = 1 \quad \checkmark$$

\therefore the point $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$
is on the unit circle.

Example 3: Determine the coordinates of all points on the unit circle with:

a) x -coordinate of $\frac{3}{5}$

Positive value \rightarrow quadrants I and IV



$$x^2 + y^2 = 1$$

$$\left(\frac{3}{5}\right)^2 + y^2 = 1$$

$$\frac{9}{25} + y^2 = 1$$

$$y^2 = \frac{25}{25} - \frac{9}{25}$$

$$\sqrt{y^2} = \sqrt{\frac{16}{25}}$$

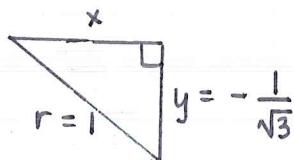
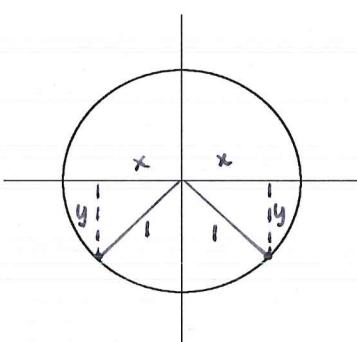
$$y = \pm \frac{4}{5}$$

So, quad I : $(\frac{3}{5}, \frac{4}{5})$

quad IV : $(\frac{3}{5}, -\frac{4}{5})$

b) y -coordinate of $-\frac{1}{\sqrt{3}}$

Negative value \rightarrow quadrants III and IV



$$x^2 + \left(-\frac{1}{\sqrt{3}}\right)^2 = 1$$

$$x^2 + \frac{1}{3} = 1$$

$$x^2 = \frac{3}{3} - \frac{1}{3}$$

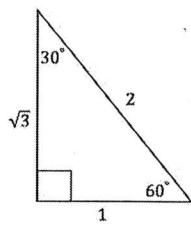
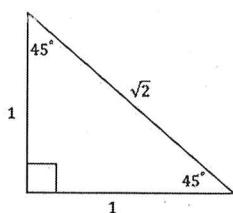
$$\sqrt{x^2} = \sqrt{\frac{2}{3}}$$

$$x = \pm \sqrt{\frac{2}{3}}$$

quad III : $(-\sqrt{\frac{2}{3}}, -\frac{1}{\sqrt{3}})$

quad IV : $(\sqrt{\frac{2}{3}}, -\frac{1}{\sqrt{3}})$

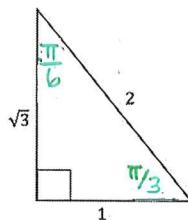
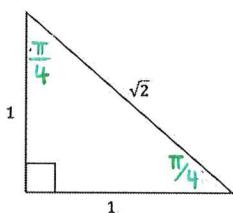
Special Triangles in degrees (from PC 11)



$$45^\circ \cdot \frac{\pi}{180^\circ} = \frac{\pi}{4}$$

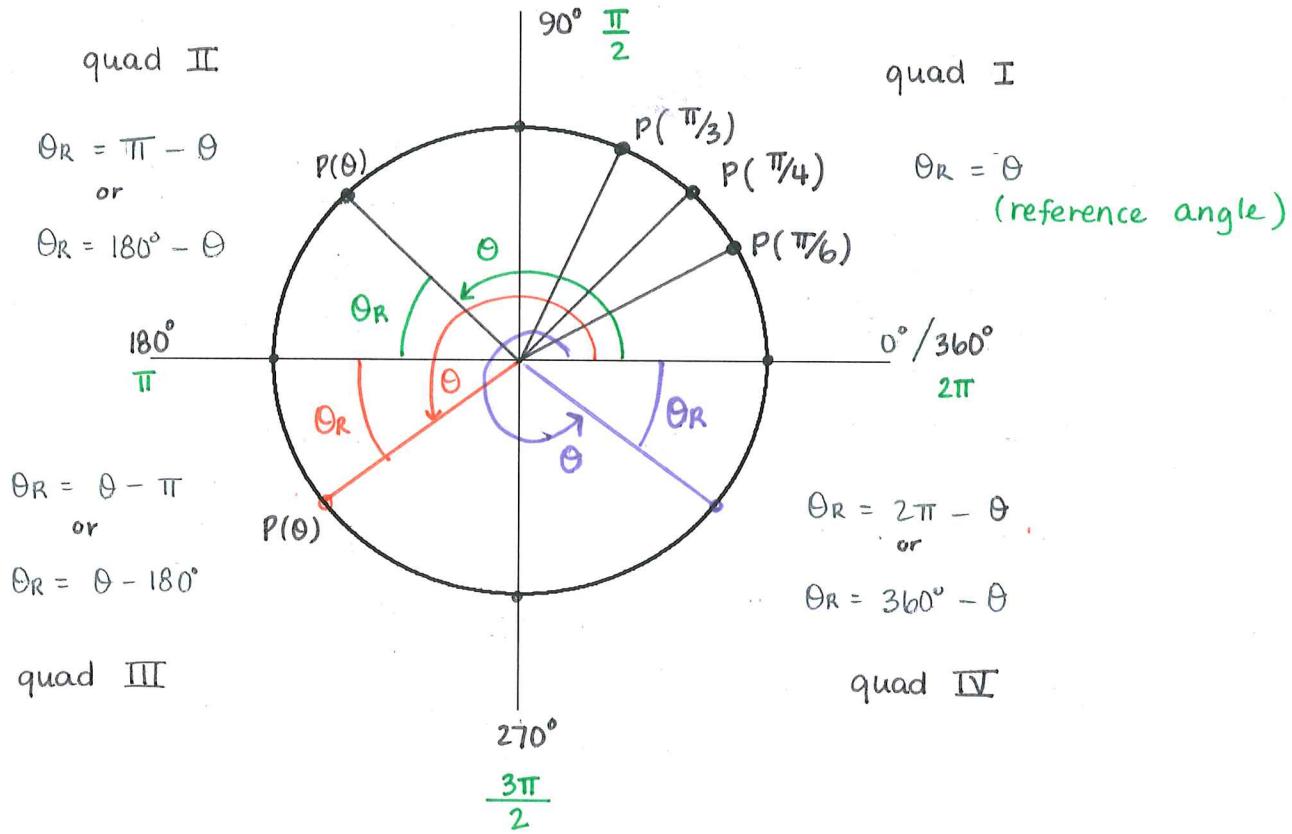
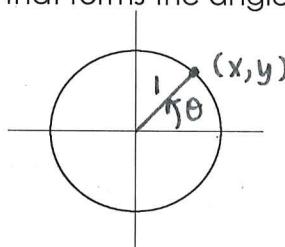
$$30^\circ \cdot \frac{\pi}{180^\circ} = \frac{\pi}{6}$$

Special Triangles in radians



$$60^\circ \cdot \frac{\pi}{180^\circ} = \frac{\pi}{3}$$

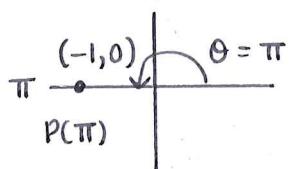
P(θ) refers to the point P on the unit circle that forms the angle θ . Since P is a point, it has the coordinates: $P(\theta) = (x, y)$



Example 3: Find the exact coordinates of: (on a unit circle, $r = 1$)

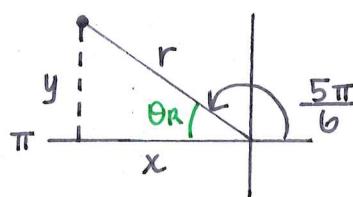
need an $r = 1$
for unit
circle so we
 \div every side by 2

a) $P(\pi)$



$$P(\pi) = (-1, 0)$$

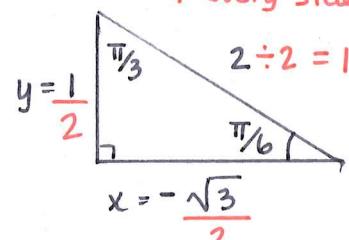
b) $P\left(\frac{5\pi}{6}\right)$



$$\theta_R = \pi - \frac{5\pi}{6}$$

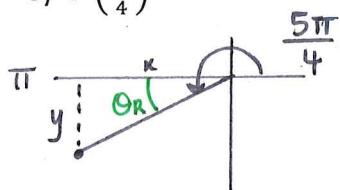
$$= \frac{6\pi}{6} - \frac{5\pi}{6}$$

$$\theta_R = \frac{\pi}{6}$$



$$P\left(\frac{5\pi}{6}\right) = \left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$$

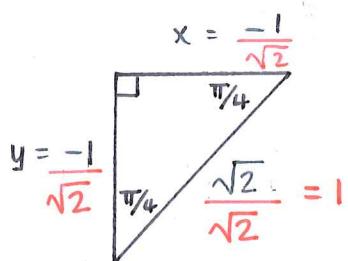
c) $P\left(\frac{5\pi}{4}\right)$



$$\theta_R = \frac{5\pi}{4} - \pi$$

$$= \frac{5\pi}{4} - \frac{4\pi}{4}$$

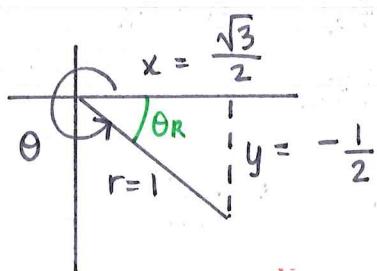
$$\theta_R = \frac{\pi}{4}$$



$$P\left(\frac{5\pi}{4}\right) = \left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$$

Example 4: Find angle θ such that $P(\theta) = \left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

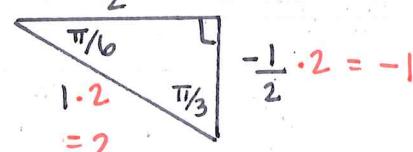
x^T y $r = 1$



$$\theta_R = \frac{\pi}{6}$$

$$\theta = 2\pi - \theta_R$$

$$\frac{\sqrt{3}}{2} \cdot 2 = \sqrt{3}$$



mult. each side by 2 to
create one of the
special Δ.

$$\theta = \frac{12\pi}{6} - \frac{\pi}{6}$$

$$\theta = \frac{11\pi}{6}$$

Practice: p.186 #1b, 2bc, 3ace, 4, 5, 9, 13