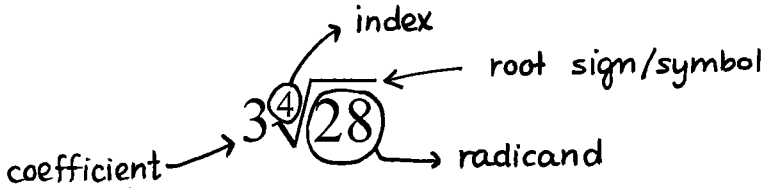
4.3 Mixed and Entire Radicals



Perfect Squares:

$2^2 =$ 4
$3^2 =$ 9
$4^2 =$ 16
$5^2 =$ 25
$6^2 =$ 36
$7^2 =$ 49
$8^2 =$ 64
$9^2 =$ 81
$10^2 =$ 100
$11^2 =$ 121
$12^2 =$ 144

Perfect Cubes:

$2^3 =$ 8
$3^3 =$ 27
$4^3 =$ 64
$5^3 =$ 125
$6^3 =$ 216

Entire Radical: A radical with a coefficient of 1. All the numbers are under the root symbol (except for the index).

ex: $\sqrt{140}$ or $\sqrt[3]{15}$

Mixed Radical: When the coefficient is anything other than 1.

ex: $3\sqrt{12}$ or $2\sqrt[4]{99}$

Example 1: Express a whole radical as a mixed radical (**simplify** the radical)

Steps : 1) Find the largest perfect square/cube that is a factor of the radicand.

$\sqrt{(\text{perfect square/cube}) \cdot (\text{another factor})}$

2) Rewrite the radical as the product of two radicals.

$(\sqrt{\text{perfect square or cube}}) \cdot (\sqrt{\text{another factor}})$

3) Simplify your answer by finding the square/cube root of the perfect square/cube.

$\text{coefficient} \cdot \sqrt{\text{another factor}}$

a) $\sqrt{12}$ (perfect square)

$= \sqrt{4 \cdot 3}$

$= \sqrt{4} \cdot \sqrt{3}$

$= 2\sqrt{3}$

b) $\sqrt{45}$

$= \sqrt{9 \cdot 5}$

$= \sqrt{9} \cdot \sqrt{5}$

$= 3 \cdot \sqrt{5}$

c) $\sqrt{72}$ (largest perfect square)

$= \sqrt{36 \cdot 2}$

$= \sqrt{36} \cdot \sqrt{2}$

$= 6\sqrt{2}$

d) $\sqrt{80}$

$= \sqrt{16 \cdot 5}$

$= \sqrt{16} \cdot \sqrt{5}$

$= 4\sqrt{5}$

e) $\sqrt[3]{24}$ (perfect cube)

$= \sqrt[3]{8 \cdot 3}$

$= \sqrt[3]{8} \cdot \sqrt[3]{3}$

$= 2\sqrt[3]{3}$

c) $\sqrt[3]{144}$

$= \sqrt[3]{8 \cdot 18}$

$= \sqrt[3]{8} \cdot \sqrt[3]{18}$

$= 2\sqrt[3]{18}$

Example 2 : Express a mixed radical as a whole radical

Steps : 1) Square or cube the coefficient out in front of the radical sign (depends on the index)

2) Then, move it under the radical and multiply it by the radicand

a) $5\sqrt{3}$

$= \sqrt{5^2 \cdot 3}$

$= \sqrt{25 \cdot 3}$

$= \sqrt{75}$

b) $2\sqrt{7}$

$= \sqrt{2^2 \cdot 7}$

$= \sqrt{4 \cdot 7}$

$= \sqrt{28}$

c) $3\sqrt[3]{4}$ (★)

$= \sqrt[3]{3^3 \cdot 4}$

$= \sqrt[3]{27 \cdot 4}$

$= \sqrt[3]{108}$

d) $2\sqrt{3}$

$= \sqrt{2^2 \cdot 3}$

$= \sqrt{4 \cdot 3}$

$= \sqrt{12}$

e) $2\sqrt[3]{5}$

$= \sqrt[3]{2^3 \cdot 5}$

$= \sqrt[3]{8 \cdot 5}$

$= \sqrt[3]{40}$

c) $2\sqrt[4]{3}$

$= \sqrt[4]{2^4 \cdot 3}$

$= \sqrt[4]{16 \cdot 3}$

$= \sqrt[4]{48}$

Practice: p. 218 # 4defgh, 5defgh, 10defgh, 11def, 12fgh