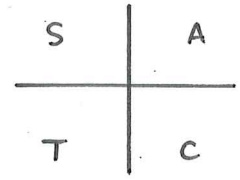
4.4 Introduction to Trig Equations

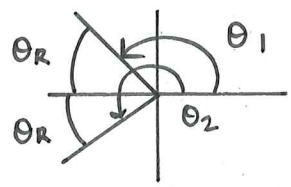


Trig Equation: An equation involving trigonometric ratios.

$$6\tan\theta + 1 = 0 \quad ; \quad \underbrace{0 \le \theta < 2\pi}_{\text{restricted domain}}$$

solve for θ

Example 1: Solve $\cos\theta = -0.437$; $0 \le \theta < 2\pi$ Express solution to nearest 3 decimal places.



$\theta = \cos^{-1}(-0.437)$

$\theta_1 = 2.023$

$\theta_R = \pi - \theta_1$

$= \pi - 2.023$

$\theta_R = 1.119$

$\theta_2 = \pi + \theta_R$

$= \pi + 1.119$

$\theta_2 = 4.260$

Example 2: Solve $-3 - 5\sin\theta = 2\sin\theta - 2$; $\underbrace{[0°,360°)}_{0° \le \theta < 360°}$ Express solution to nearest 3 decimal places.

$-5\sin\theta = 2\sin\theta + 1$

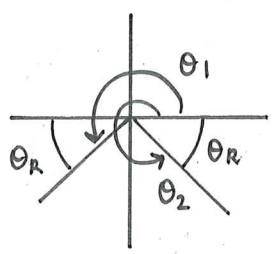
$-7\sin\theta = 1$

$\sin\theta = \frac{-1}{7}$

$\theta = \sin^{-1}\left(\frac{1}{7}\right)$

$= -8.213°$

$\theta_R = 8.213°$



$\theta_1 = 180° + \theta_R$

$= 180° + 8.213°$

$\theta_1 = 188.213$

$\theta_2 = 360° - \theta_R$

$= 360° - 8.213°$

$\theta_2 = 351.787°$

Example 3: Solve using exaxt values. $2\sec\theta = 4$; $0 \le \theta < 2\pi$

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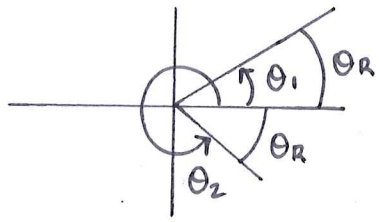
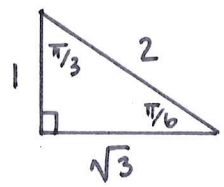
$2\sec\theta = 4$

$\sec\theta = \frac{4}{2}$

$\sec\theta = \frac{2}{1}$

$\cos\theta = \frac{1}{2}$





$\theta_R = \frac{\pi}{3}$

$\theta_1 = \theta_R$

$\theta_1 = \frac{\pi}{3}$

$\theta_2 = 2\pi - \theta_R$

$= 2\pi - \frac{\pi}{3}$

$= \frac{6\pi}{3} - \frac{\pi}{3}$

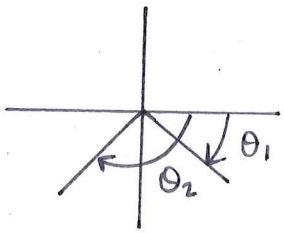
$\theta_2 = \frac{5\pi}{3}$

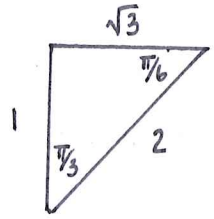
Example 4: Solve using exact values. $3 + 5\csc\theta = 1 + 4\csc\theta$; $-\pi \le \theta < \pi$

$3 + 5\csc\theta = 1 + 4\csc\theta$

$\csc\theta = \frac{-2}{1}$

$\sin\theta = -\frac{1}{2}$





$\theta_R = \frac{\pi}{6}$

$\theta_1 = -\theta_R$

$\theta_1 = -\frac{\pi}{6}$

$\theta_2 = -(\pi - \theta_R)$

$= -\left(\frac{6\pi}{6} - \frac{\pi}{6}\right)$

$\theta_2 = -\frac{5\pi}{6}$

Example 5: Solve using exact values $4\sin^2\theta = 3$ $[0°, 360°)$

$0° \le \theta < 360°$

$4\sin^2\theta = 3$

$\sqrt{\sin^2\theta} = \sqrt{\frac{3}{4}}$

$\sin\theta = \pm\sqrt{\frac{3}{4}}$

$\sin\theta = \pm\frac{\sqrt{3}}{2}$

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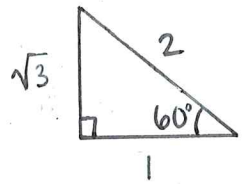
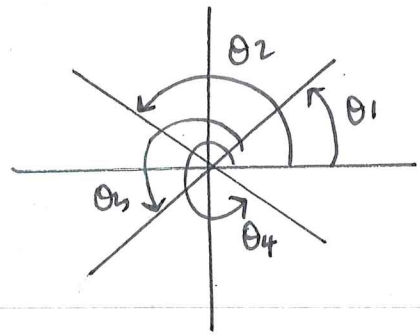


$\theta_R = 60°$



$\theta_1 = \theta_R$

$\theta_1 = 60°$

$\theta_2 = 180° - 60°$

$\theta_2 = 120°$

$\theta_3 = 180° + 60°$

$\theta_3 = 240°$

$\theta_4 = 360° - 60°$

$\theta_4 = 300°$

$\sin\theta = \frac{\sqrt{3}}{2}$, $\sin\theta = -\frac{\sqrt{3}}{2}$

2 solutions 2 solutions.

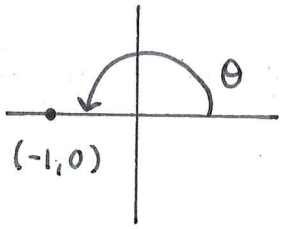
General Solutions: A general solution to a trig equation is a statement that represents all possible solutions (no restricted domains).

Example 6: Find the general solution, in radians, using exact values: $\cos\theta + 1 = 0$.

$\cos\theta + 1 = 0$

$\cos\theta = \frac{-1}{1}$ $\frac{x}{r}$

$\theta = \pi$



general solutions → create coterminal formula!

$\theta = \pi \pm 2\pi n$, $n \in N$

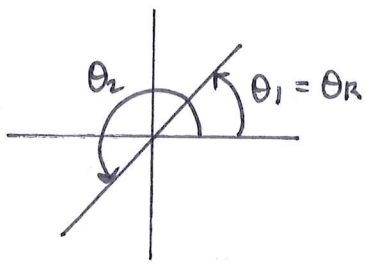
or

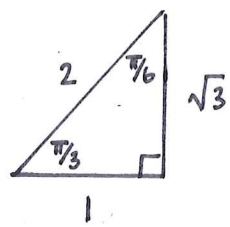
$\theta = \pi(1 + 2n)$, $n \in N$

Example 7: Find the general solution, in radians, using exact values: $\tan\theta - \sqrt{3} = 0$.

$\tan\theta - \sqrt{3} = 0$

$\tan\theta = \dfrac{\sqrt{3}}{1}$





$\theta_R = \dfrac{\pi}{3}$

$\theta_1 = \theta_R$

$\theta_1 = \dfrac{\pi}{3}$

$\theta_2 = \pi + \dfrac{\pi}{3}$

$= \dfrac{3\pi}{3} + \dfrac{\pi}{3}$

$\theta_2 = \dfrac{4\pi}{3}$

general solution :

$\theta = \dfrac{\pi}{3} \pm 2\pi n, \; n \in N$

$\theta = \dfrac{4\pi}{3} \pm 2\pi n, \; n \in N$

or

$\theta = \dfrac{\pi}{3} \pm \pi n, \; n \in N$

Practice: p.211 #2, 3, 4abef, 5, 6ace, 7bc, 18