5.1 Graphing Sine and Cosine Functions

The Graph of $f(\theta) = \sin\theta$

$\sin\theta = \frac{y}{r}$

Unit Circle

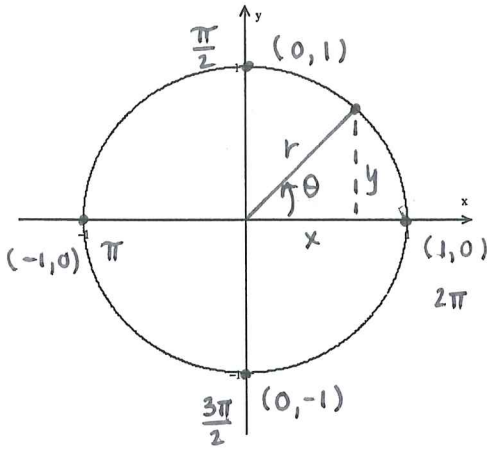
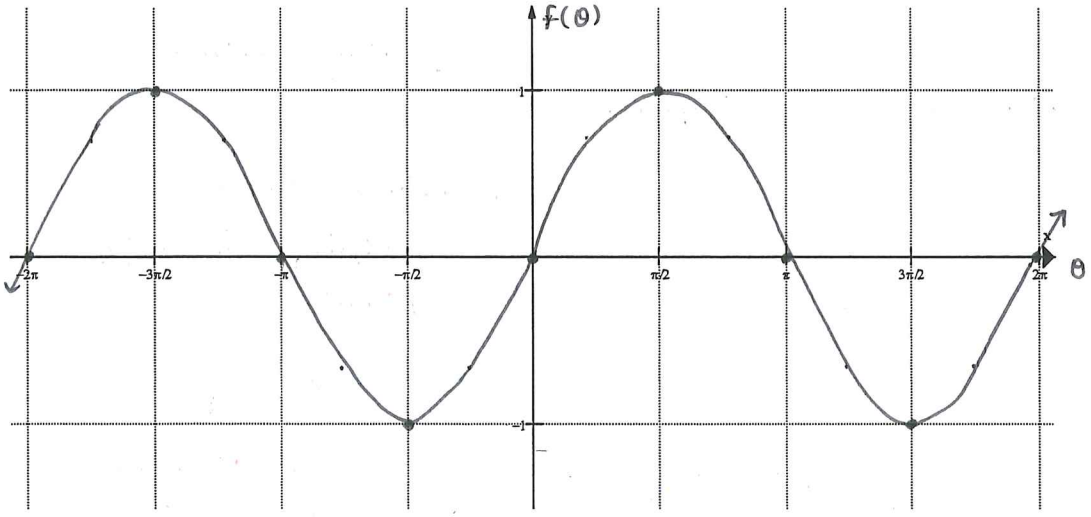


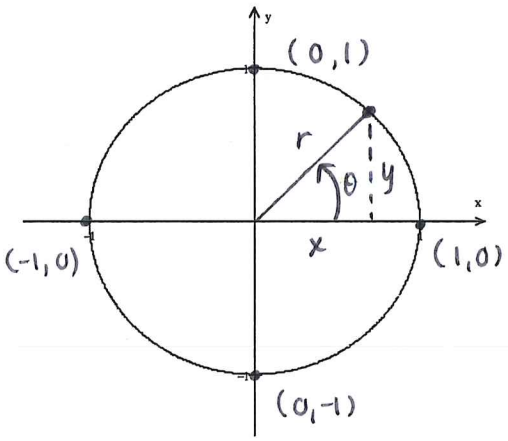
Table of Values

θ	$f(\theta)$	
0	0	$\frac{0}{1}$
$\frac{\pi}{2}$	1	$\frac{1}{1}$
π	0	$\frac{0}{1}$
$\frac{3\pi}{2}$	-1	$\frac{-1}{1}$
2π	0	$\frac{0}{1}$



The Graph of $f(\theta) = \cos\theta$

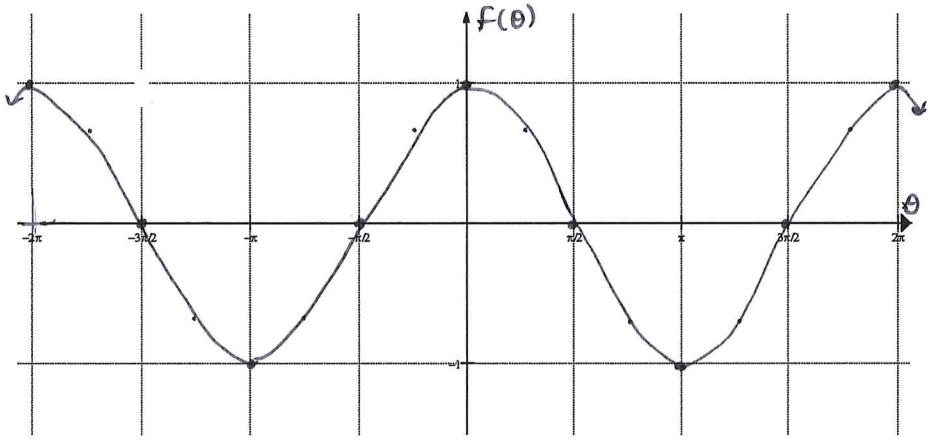
Unit Circle



$\cos\theta = \frac{x}{r}$

Table of Values

θ	$f(\theta)$	
0	1	$\frac{1}{1}$
$\frac{\pi}{2}$	0	$\frac{0}{1}$
π	-1	$\frac{-1}{1}$
$\frac{3\pi}{2}$	0	$\frac{0}{1}$
2π	1	$\frac{1}{1}$



Functions that repeat themselves over a particular interval are called periodic functions. The interval is called the period. The amplitude of a periodic function is one half the difference between the maximum and minimum values.

$$Amplitude = \frac{|max - min|}{2}$$

absolute value (amplitude is always positive)

Example 1: What is the amplitude and period of $y = \sin\theta$ and $y = \cos\theta$?

max = 1

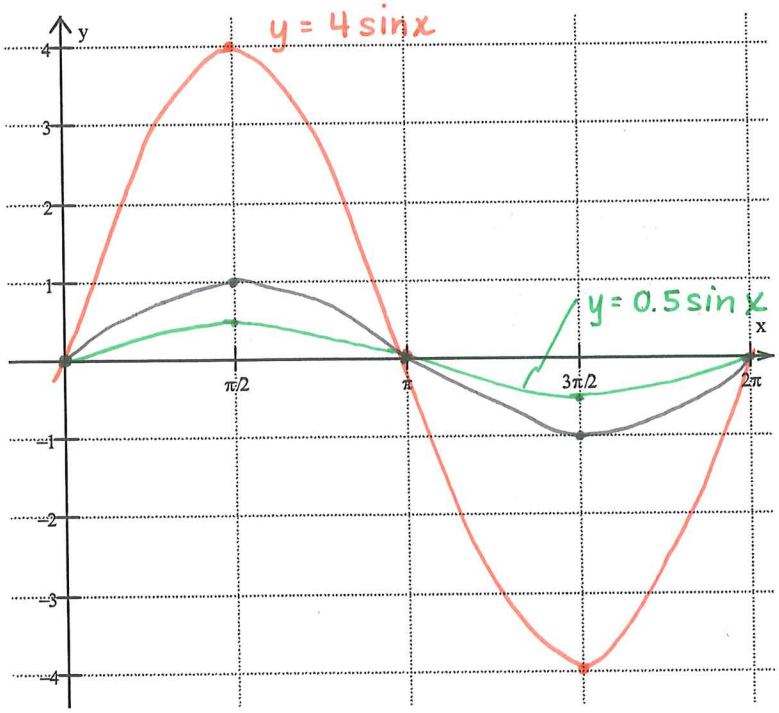
min = -1

$Amp = \frac{|1-(-1)|}{2} = \frac{2}{2} = 1$ } for both functions

period = 2π } for both functions

one complete cycle

Example 2: Graph of $y = \sin x$, $y = 4\sin x$, and $y = 0.5\sin x$ for $0 \le x \le 2\pi$ on the same axes and state the amplitude of each function.



$y = \sin x$ ("average joe")

amp = 1

$y = 4\sin x$

x	y · 4
0	0
π/2	4
π	0
3π/2	−4
2π	0

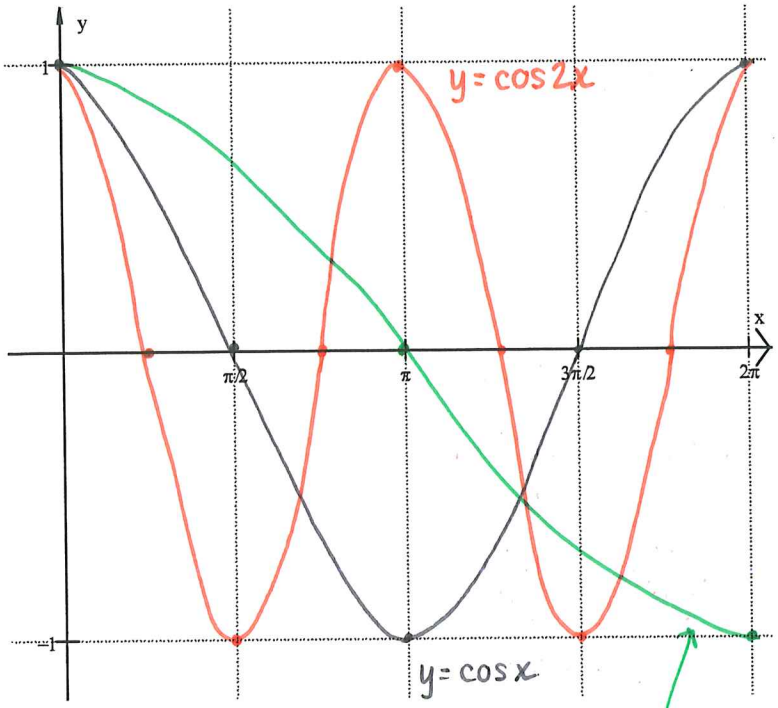
amplitude = 4

$y = 0.5\sin x$

x	y · 0.5
0	0
π/2	0.5
π	0
3π/2	−0.5
2π	0

amplitude = 0.5

Example 3: Graph $y = \cos x$, $y = \cos 2x$, and $y = \cos\frac{x}{2}$ for $0 \le x \le 2\pi$ on the same axes and state the period of each function.



$y = \cos x$ ("average joe")

period = 2π

$y = \cos 2x$

·½ x	y
0 0	1
π/4 π/2	0
π/2 π	−1
3π/4 3π/2	0
π 2π	1

period = π

$y = \cos\frac{x}{2}$

2· x	y
0 0	1
π π/2	0
2π π	−1
3π 3π/2	0
4π 2π	1

period = 4π

$y = \cos\frac{x}{2}$

(only half one cycle shown)

Note: For functions of the form $y = a\sin bx$ and $y = a\cos bx$, where $a, b \neq 0$, the amplitude is $|a|$ and the period is $\frac{2\pi}{b}$ or $\frac{360^\circ}{b}$

Example 4: State the amplitude and period of the following functions in radians:

a) $y = 2\cos\frac{x}{3}$ → $y = 2\cos\frac{1}{3}x$ (2 = a, $\frac{1}{3}$ = b)

amplitude = $|2| = 2$

period = $\frac{2\pi}{b}$

$= \frac{2\pi}{(1/3)}$

$= 2\pi \cdot 3$

$= 6\pi$

b) $y = -4\sin 4x$

amp = $|-4| = 4$

period = $\frac{2\pi}{4}$

$= \frac{\pi}{2}$

Example 5: State the amplitude and period of the following functions in degrees:

a) $y = \frac{1}{2}\sin 2\theta$

amp = $\left|\frac{1}{2}\right| = \frac{1}{2}$

period = $\frac{360^\circ}{2}$

$= 180^\circ$

b) $y = -\cos\frac{2\theta}{3}$

amp = $|-1| = 1$

per = $\frac{360^\circ}{2/3}$

$= 360^\circ \times \frac{3}{2}$

$= 540^\circ$

Example 6: Write an equation of the given function with the following characteristics:

a) sine function

amplitude = 3 and *period* = $\frac{\pi}{4}$

$y = a\sin bx$

$a = 3$

period = $\frac{2\pi}{b}$

$b \cdot \frac{\pi}{4} = \frac{2\pi}{b} \cdot b$

$4 \cdot \frac{b\pi}{4} \cdot 2\pi \cdot 4$

$b = 8$

$y = 3\sin 8x$

b) cosine function

amplitude = $\frac{1}{3}$ and *period* = 60°

$y = a\cos bx$

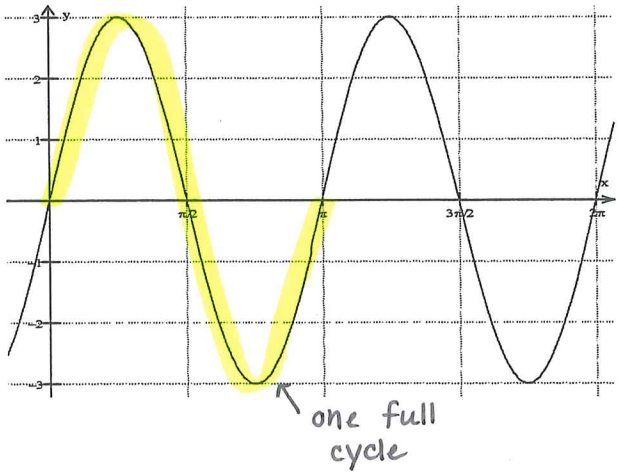
$b \cdot 60^\circ = \frac{360^\circ}{b} \cdot b$

$\frac{60^\circ b}{60^\circ} = \frac{360^\circ}{60^\circ}$

$b = 6$

$y = \frac{1}{3}\cos 6x$

Example 7: Determine the equation for the following sine function. $y = a\sin bx$



$$\text{amp} = \frac{|\max - \min|}{2} = \frac{|3-(-3)|}{2} = 3$$

$a = 3$

$\text{per} = \pi$

$b \cdot \pi = \frac{2\pi}{b} \cdot b$

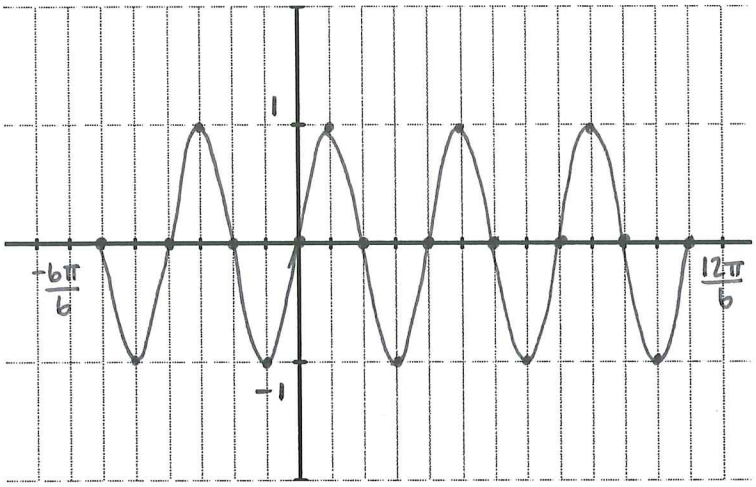
$b\pi = 2\pi$

$b = 2$

$y = 3\sin 2x$

Example 8: Sketch the graph of the following functions.

a) $y = \sin 3x, -\pi \le x \le 2\pi$ $\quad \frac{-b\pi}{b} \le x \le \frac{12\pi}{b}$



$\text{amp} = 1$

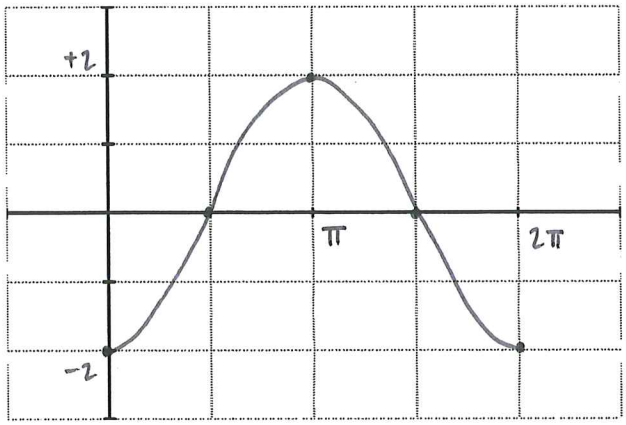
$\text{period} = \frac{2\pi}{3} = \frac{4\pi}{6}$

(easier to graph if increments are by $\frac{\pi}{6}$)

$\cdot\frac{1}{3}$ x	y	
0 ~~0~~	0	start
$\pi/6$ ~~$\pi/2$~~	1	
$\pi/3 = 2\pi/6$ ~~π~~	0	middle
$3\pi/6$ ~~$3\pi/2$~~	-1	
$2\pi/3 = 4\pi/6$ ~~2π~~	0	end

} one cycle

b) $y = -2\cos x, 0 \le x \le 2\pi$



$\text{amp} = |-2| = 2$

a is neg → reflection over x-axis

$\text{period} = 2\pi$

x	$y \cdot -2$
0	+ -2
$\pi/2$	~~0~~ 0
π	-+ 2
$3\pi/2$	~~0~~ 0
2π	+ -2

Practice: p.233 #1, 2, 4 – 11, 14, 15, 20