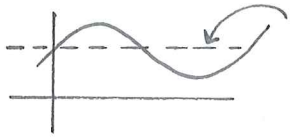
5.2 Transformations of Sinusoidal Functions

The principles of transformations from Chapter 1 can be applied to the trigonometric functions.

Equation	What it was called	*Sin/Cos* equivalent	What it does...
$y = af(x)$	vertical stretch and/or reflection over x-axis	$y = a\sin x$ or $y = a\cos x$	change in amplitude and/or reflection over x-axis
$y = f(bx)$	horizontal stretch and/or reflection over y-axis	$y = \sin(bx)$ or $y = \cos(bx)$	change of period period = $\frac{2\pi}{b}$ or $\frac{360^\circ}{b}$
$y = f(x) + k$	vertical translation	$y = \sin x + k$ or $y = \cos x + k$	vertical displacement  middle line = k
$y = f(x - h)$	horizontal translation	$y = \sin(x-h)$ or $y = \cos(x-h)$	phase shift $(x-h) \rightarrow$ $(x+h) \leftarrow$

Any sine or cosine function can be expressed in the form:

$$y = a\sin b(x-h) + k \quad \text{or} \quad y = a\cos b(x-h) + k$$

Amplitude = $|a|$

Period = $\frac{2\pi}{b}$ or $\frac{360^\circ}{b}$

Phase Shift = h

Vertical Displacement (new middle line) = k

Example 1: A sine function is given by the equation $y = 3\sin 2\left(x - \frac{\pi}{4}\right) + 2$. Determine the following:

a b h K

a) Amplitude

$|3| = 3$

b) Period

$\frac{2\pi}{b} = \frac{2\pi}{2} = \pi$

b) Phase shift

$h = \frac{\pi}{4}$ (right)

d) Vertical displacement

$K = 2$ (up)

e) Domain

$\{ x \mid x \in \mathbb{R} \}$

f) Range

min : k - amp $\quad 2 - 3 = -1$ $\qquad$ max : amp + k $\quad 3 + 2 = 5$

$\{ y \mid -1 \le y \le 5, y \in \mathbb{R} \}$

g) y-intercept (when $x = 0$)

$y = 3\sin 2\left(0 - \frac{\pi}{4}\right) + 2$

$= 3\sin 2\left(-\frac{\pi}{4}\right) + 2$

$= 3\sin\left(-\frac{\pi}{2}\right) + 2$

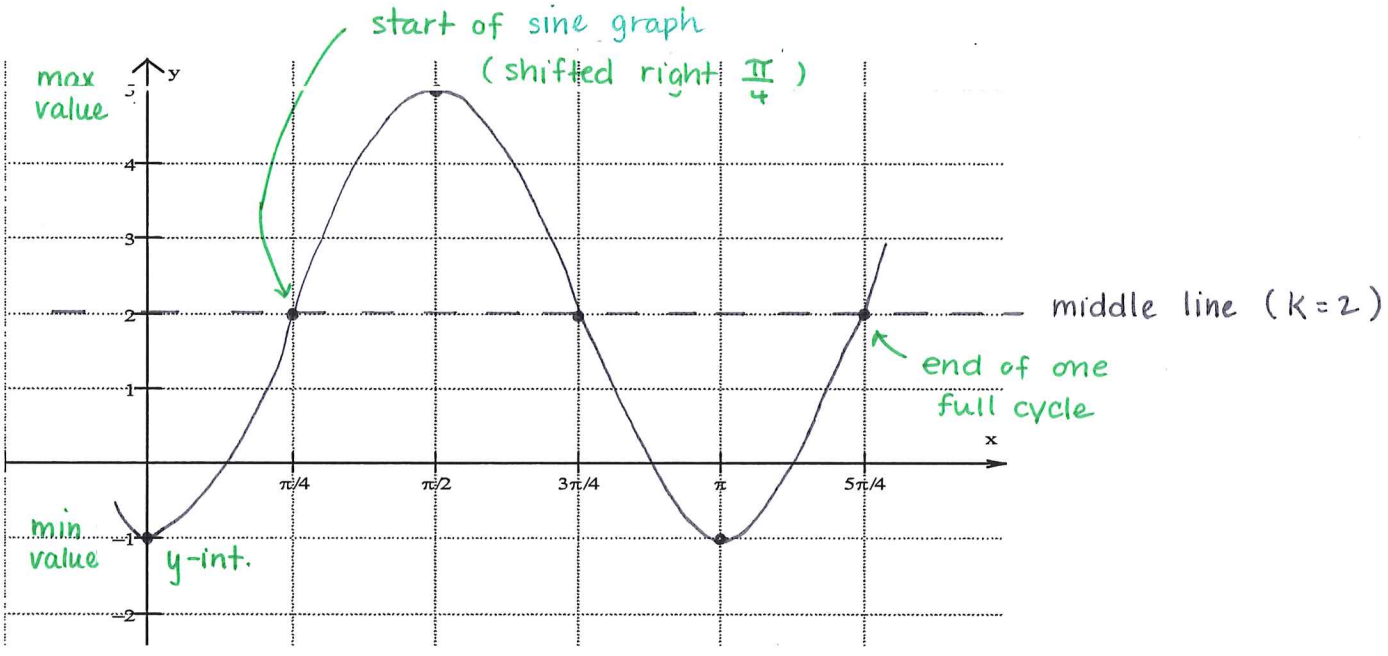
$= 3(-1) + 2$

$= -3 + 2$

$= -1$

h) Sketch the graph

(h) +π/4	(1/b) ·1/2 x	y	(a) ·3	(k) +2
π/4	0	0	0	2
π/2	π/4	1	3	5
3π/4	π/2	0	0	2
π	3π/4	−1	−3	−1
5π/4	π	0	0	2
	2π			



Example 2: A cosine function is given by the equation $y = -2\cos\frac{2}{3}(x - 45°) + 1$. Determine the following:

a) Amplitude

$|-2| = 2$

b) Period

$\frac{360°}{b} = \frac{360°}{2/3} = 360° \cdot \frac{3}{2} = 540°$

b) Phase shift

$h = 45°$ (right)

d) Vertical displacement

$K = 1$ (up)

e) Domain

$\{ x \mid x \in \mathbb{R} \}$

f) Range

min: K-amp $= 1 - 2 = -1$ max: amp + K $= 2 + 1 = 3$

$\{ y \mid -1 \leq y \leq 3, \ y \in \mathbb{R} \}$

g) y-intercept

$y = -2\cos\frac{2}{3}(0 - 45°) + 1$

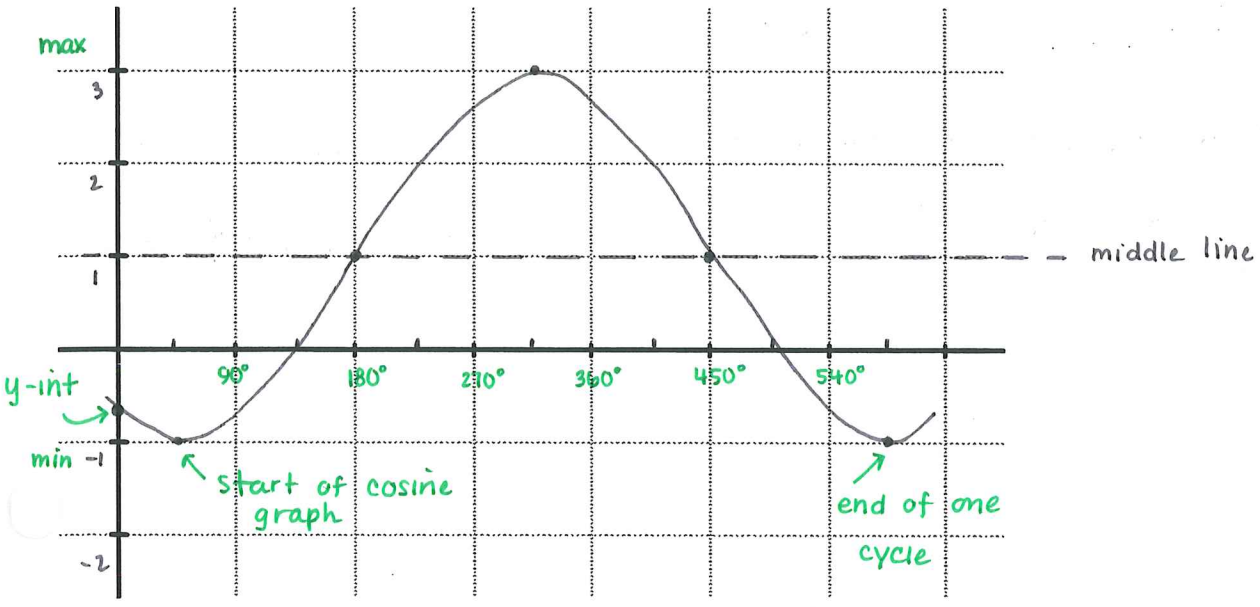
$y = -2\cos(-30°) + 1$

$y = -2(0.8660) + 1$

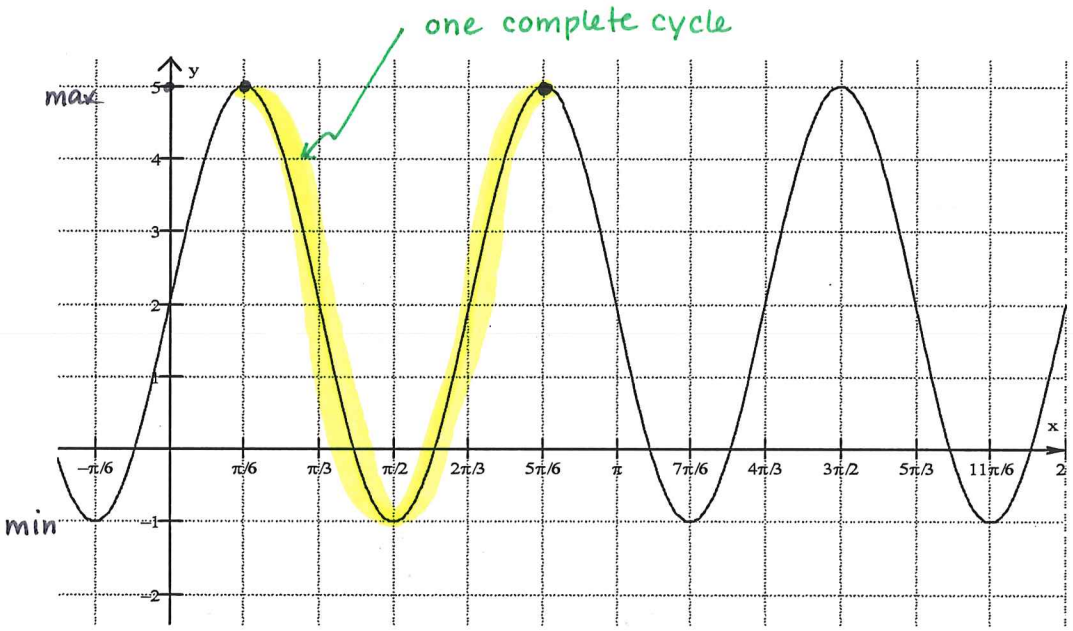
$y \simeq -0.732$

h) Sketch the graph

(h) +45°	(1/b) ·3/2	x	y	(a) ·−2	(K) +1
45°	0	0	+	−2	−1
180°	135°	90°	0	0	1
315°	270°	180°	−1	2	3
450°	405°	270°	0	0	1
585°	540°	360°	+	−2	−1



Example 3: The partial graph of a cosine function is shown. Determine the equation of the function in the form $y = A\cos B(x - C) + D$



A : amplitude

$$\text{Amp} = \frac{|\text{max} - \text{min}|}{2}$$

$$= \frac{|5 - (-1)|}{2}$$

$$A = 3$$

B : period

$$\frac{5\pi}{6} - \frac{\pi}{6} = \frac{4\pi}{6} = \frac{2\pi}{3}$$

$$\text{period} = \frac{2\pi}{B}$$

$$\cancel{3}B \cdot \frac{2\pi}{\cancel{3}} = \frac{2\pi}{B} \cdot \cancel{B} \cdot 3$$

$$\frac{B \cdot \cancel{2\pi}}{\cancel{2\pi}} = \frac{\cancel{2\pi} \cdot 3}{\cancel{2\pi}}$$

$$B = 3$$

C : phase shift

shifted right $\frac{\pi}{6}$

$$C = \frac{\pi}{6}$$

D : vert displacement (middle line)

$$D = \text{max} - \text{amp}$$

$$= 5 - 3$$

$$D = 2$$

$$y = A\cos B(x - C) + D$$

$$y = 3\cos 3\left(x - \frac{\pi}{6}\right) + 2$$

Practice: p.250 #1bcf, 2abef, 3a i & ii, 3b, 4 – 7, 10a, 11ab, 12ac, 13, 15, 16, 27ab