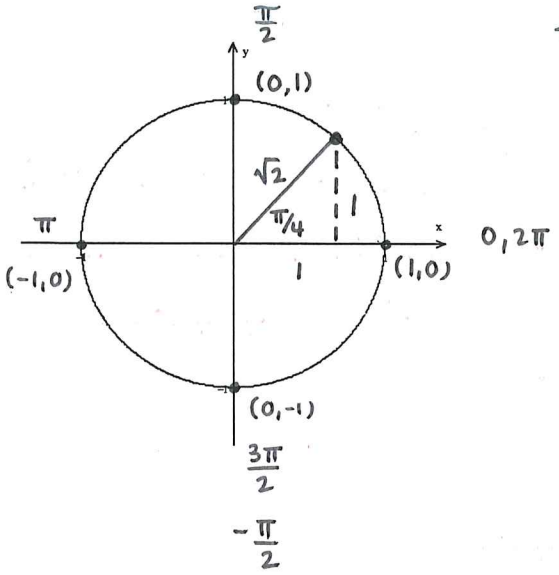
5.3 The Tangent Function

The Graph of $f(\theta) = \tan\theta$

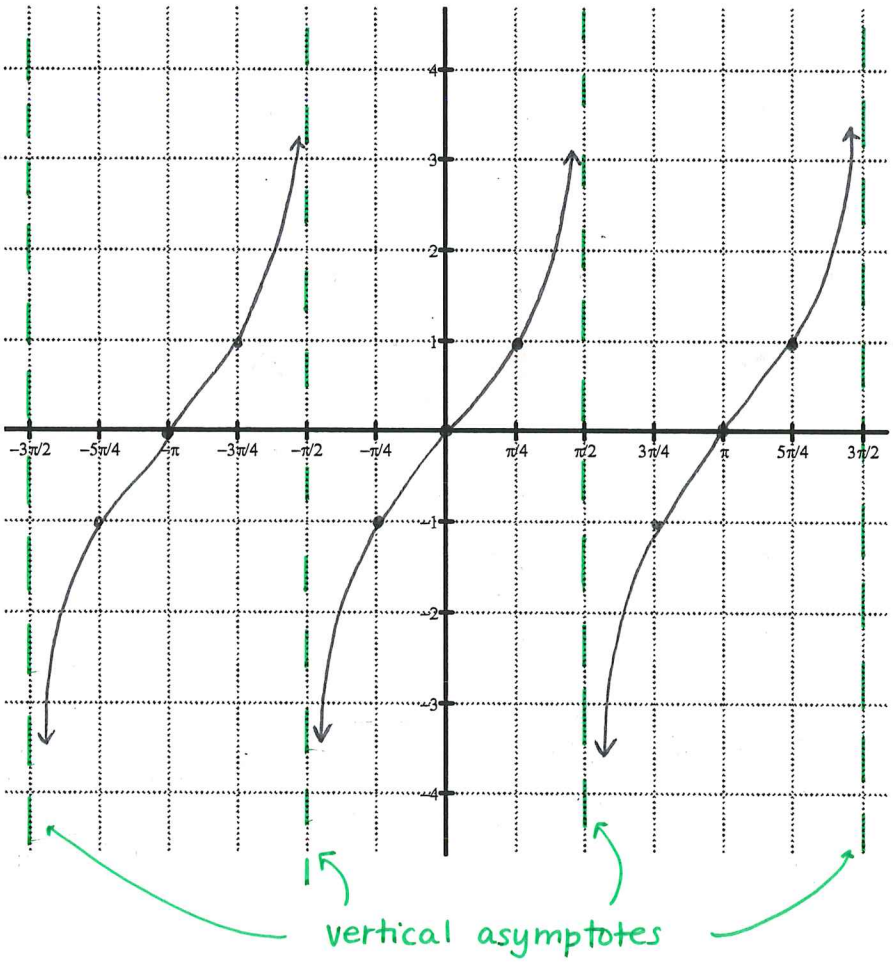
Unit circle



$\tan\theta = \frac{y}{x}$

Table of Values

θ	$f(\theta)$
$-\frac{\pi}{2}$	$\frac{-1}{0}$ = undefined
$-\frac{\pi}{4}$	$\frac{-1}{1} = -1$
0	$\frac{0}{1} = 0$
$\frac{\pi}{4}$	$\frac{1}{1} = 1$
$\frac{\pi}{2}$	$\frac{1}{0}$ = undefined



Recall : $\sin\theta = \frac{y}{r}$ $\cos\theta = \frac{x}{r}$ $\tan\theta = \frac{y}{x}$

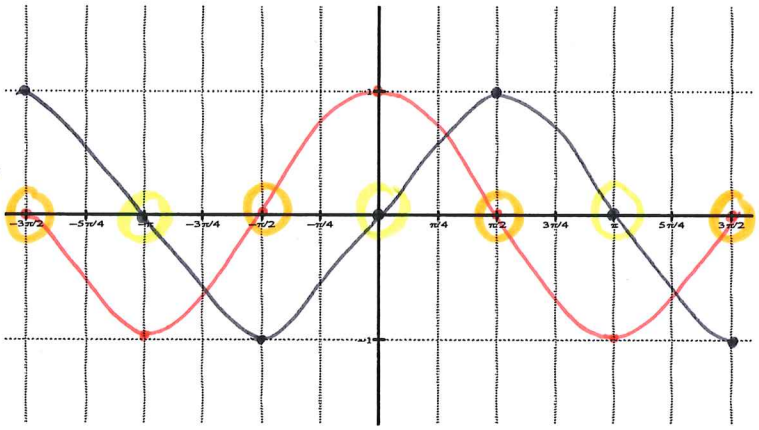
So... $\frac{\sin\theta}{\cos\theta} = \frac{\frac{y}{r}}{\frac{x}{r}} = \frac{y}{r} \cdot \frac{r}{x} = \frac{y}{x}$ ← same!

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

Graph $y = \sin\theta$ and $y = \cos\theta$ on the following graph.

$y = \sin\theta$

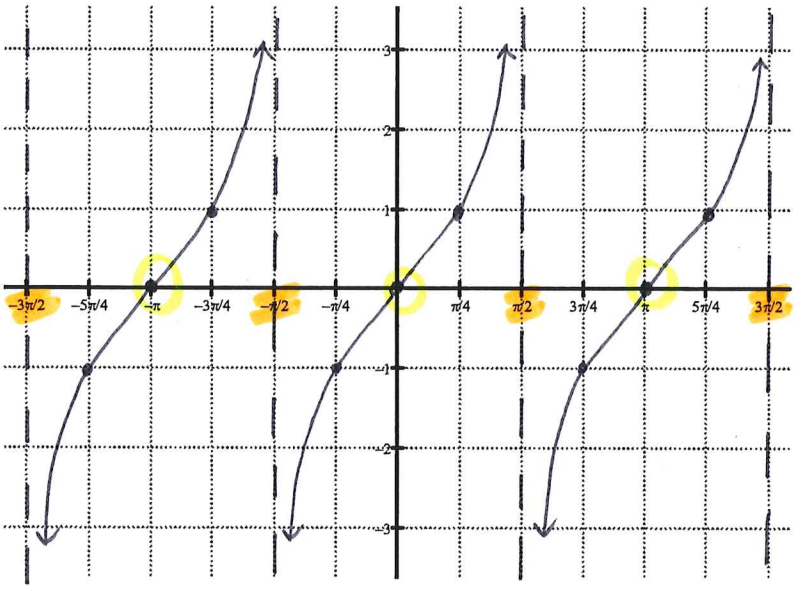
when $\sin\theta = 0$, $\tan\theta = 0$



$y = \cos\theta$

when $\cos\theta = 0$, $\tan\theta$ is undefined (there is a vertical asymptote)

Wherever $\cos\theta$ has a zero then $\tan\theta$ will have an asymptote and wherever $\sin\theta$ has a zero then $\tan\theta$ will have a zero.



What is the period of $y = \tan x$?

period $= \frac{3\pi}{2} - \frac{\pi}{2} = \frac{2\pi}{2} = \pi$

period $= \frac{\pi}{b}$

What is the amplitude of $y = \tan x$?

No max or min

→ No amplitude!

What is the value of any asymptotes of $y = \tan x$?

$$x = \frac{-3\pi}{2}, \frac{-\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}$$

etc..

What is the general equation for the asymptotes?

$x = (\text{odd \#})\left(\frac{\pi}{2}\right)$ ← half the period (unless there's a phase shift)

↑ $\tan\theta$ is undefined when $\cos\theta = 0$

$x = (2n+1)\left(\frac{\pi}{2}\right)$, $n \in I$ (↑ integer)

What is the domain?

all values of x except asymptotes

$$\{x \mid x \neq (2n+1)\left(\frac{\pi}{2}\right),\ n \in I,\ x \in \mathbb{R}\}$$

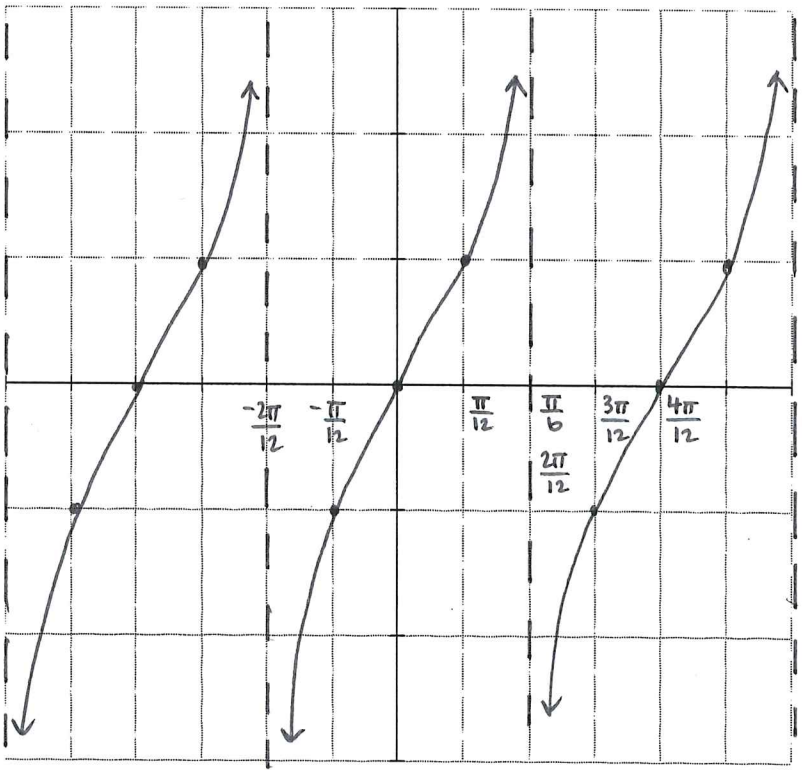
What is the range?

$$\{y \mid y \in \mathbb{R}\}$$

Example 1: Graph $y = \tan 3\theta$

$(\frac{1}{6})$

$\cdot\frac{1}{3}$ x		y
$-\frac{\pi}{6}$	~~$-\frac{\pi}{2}$~~	undef.
$-\frac{\pi}{12}$	~~$-\frac{\pi}{4}$~~	-1
0	~~0~~	0
$\frac{\pi}{12}$	~~$\frac{\pi}{4}$~~	1
$\frac{\pi}{6}$	~~$\frac{\pi}{2}$~~	undef.



period $= \frac{\pi}{b} = \frac{\pi}{3}$ ✓

period (graph) $= \frac{\pi}{6} - \left(\frac{-\pi}{6}\right) = \frac{2\pi}{6} = \frac{\pi}{3}$ ✓

equation of any asymptotes:

$x = (2n+1)\left(\frac{\pi}{b}\right)$, $n \in N$

5.3 The Tangent Function – Homework

1. Graph each function and state the following:
   - The period
   - General equation of the asymptotes
   - Domain
   - Range

a) $y = \tan 2\theta\ ;\ -\pi < \theta < \pi$

b) $y = \tan 4\theta\ ;\ -\pi < \theta < \pi$

c) $y = 3\tan\theta\ ;\ -\pi < \theta < \pi$

d) $y = \tan\left(\theta - \frac{\pi}{4}\right)\ ;\ -\pi < \theta < \pi$