5.4 Solving Trigonometric Equations Algebraically – Part 1

Example 1: Solve $4\sin\left(x - \frac{\pi}{3}\right) = 2$ using exact values for $0 \le x < 2\pi$

① Substitute θ for the angle / phase shift

let $\theta = x - \frac{\pi}{3}$

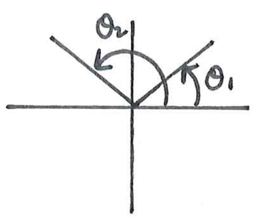
$4\sin\theta = 2$

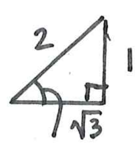
② Isolate the trig function

$$\frac{4\sin\theta}{4} = \frac{2}{4}$$

$$\sin\theta = \frac{1}{2}$$

③ Solve for θ (use special Δ's)





$\theta_R = \frac{\pi}{6}$

$\theta_1 = \theta_R$

$\theta_1 = \frac{\pi}{6}$

$\theta_2 = \pi - \theta_R$

$= \frac{6\pi}{6} - \frac{\pi}{6}$

$\theta_2 = \frac{5\pi}{6}$

④ Sub θ to solve for x

$x_1 - \frac{\pi}{3} = \theta_1$

$x_1 = \frac{\pi}{6} + \frac{2\pi}{6}$

$= \frac{3\pi}{6}$

$x = \frac{\pi}{2}$

$x_2 - \frac{\pi}{3} = \theta_2$

$x_2 = \frac{5\pi}{6} + \frac{2\pi}{6}$

$x_2 = \frac{7\pi}{6}$

Example 2: Solve $2\cos 3x = \sqrt{3}$ using exact values for $0 \le x < 2\pi$

① Substitute θ for the angle/period change

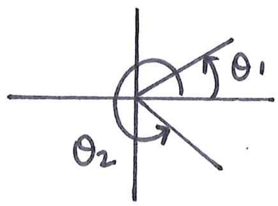
let $\theta = 3x$

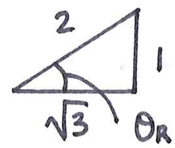
$2\cos\theta = \sqrt{3}$

② & ③ Isolate the trig function and solve for θ (use special Δ's)

$$\frac{2\cos\theta}{2} = \frac{\sqrt{3}}{2}$$

$$\cos\theta = \frac{\sqrt{3}}{2}$$





$\theta_R = \frac{\pi}{6}$

$\theta_1 = \theta_R$

$\theta_1 = \frac{\pi}{6}$

$\theta_2 = 2\pi - \theta_R$

$= \frac{12\pi}{6} - \frac{\pi}{6}$

$\theta_2 = \frac{11\pi}{6}$

④ Sub θ to solve for x

$3x_1 = \theta_1$

$$\frac{3x_1}{3} = \frac{\frac{\pi}{6}}{3}$$

$x_1 = \frac{\pi}{6} \cdot \frac{1}{3}$

$x_1 = \frac{\pi}{18}$

$3x_2 = \theta_2$

$3x_2 = \frac{11\pi}{6}$

$x_2 = \frac{11\pi}{6} \cdot \frac{1}{3}$

$x_2 = \frac{11\pi}{18}$

⑤ Add on period to find other solutions

period $= \frac{2\pi}{b} = \frac{2\pi}{3} = \frac{12\pi}{18}$

$2\pi = \frac{36\pi}{18}$ } upper limit of our restriction

$x_3 = \frac{\pi}{18} + \frac{12\pi}{18}$

$x_3 = \frac{13\pi}{18}$

$x_4 = \frac{11\pi}{18} + \frac{12\pi}{18}$

$x_4 = \frac{23\pi}{18}$

$x_5 = \frac{13\pi}{18} + \frac{12\pi}{18}$

$x_5 = \frac{25\pi}{18}$

$x_6 = \frac{23\pi}{18} + \frac{12\pi}{18}$

$x_6 = \frac{35\pi}{18}$

Example 3: Find the general solution, in radians, using exact values : $\sqrt{2}\cos 2\left(x+\frac{\pi}{3}\right)+5=4$

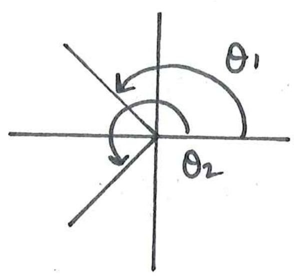
Follow steps 1-4 similar to Example 1.

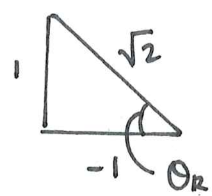
let $\theta = 2\left(x+\frac{\pi}{3}\right)$

$\sqrt{2}\cos\theta + 5 = 4$

$\sqrt{2}\cos\theta = -1$

$\cos\theta = \frac{-1}{\sqrt{2}}$





$\theta_R = \frac{\pi}{4}$

$\theta_1 = \pi - \theta_R$

$= \frac{4\pi}{4} - \frac{\pi}{4}$

$\theta_1 = \frac{3\pi}{4}$

$\theta_2 = \pi + \theta_R$

$= \frac{4\pi}{4} + \frac{\pi}{4}$

$\theta_2 = \frac{5\pi}{4}$

$2\left(x_1 + \frac{\pi}{3}\right) = \theta_1$

$\frac{2\left(x_1 + \frac{\pi}{3}\right)}{2} = \frac{\frac{3\pi}{4}}{2}$

$x_1 + \frac{\pi}{3} = \frac{3\pi}{8}$

$x_1 = \frac{9\pi}{24} - \frac{8\pi}{24}$

$x_1 = \frac{\pi}{24}$

$2\left(x_2 + \frac{\pi}{3}\right) = \theta_2$

$\frac{2\left(x_2 + \frac{\pi}{3}\right)}{2} = \frac{\frac{5\pi}{4}}{2}$

$x_2 + \frac{\pi}{3} = \frac{5\pi}{8}$

$x_2 = \frac{15\pi}{24} - \frac{8\pi}{24}$

$x_2 = \frac{7\pi}{24}$

★ General Solution :

Add on period (n)

period : $\frac{2\pi}{2} = \pi$

$x = \frac{\pi}{24} + \pi n$

$x = \frac{7\pi}{24} + \pi n$

$n \in I$

integer

Example 4: Solve $2\sin^2 x - 3\sin x - 2 = 0\,;\, 0 \le x < 2\pi$. Express your answer accurate to 2 decimal places.

This takes the form of a quadratic equation.

let $m = \sin x$

$2m^2 - 3m - 2 = 0$ $\qquad \underline{-4} \times \underline{1} = -4$

$\qquad\qquad\qquad\qquad\quad \underline{-4} + \underline{1} = -3$

$2m^2 - 4m + m - 2 = 0$

$2m(m-2) + 1(m-2) = 0$

$(m-2)(2m+1) = 0$

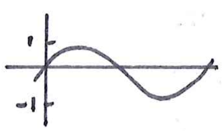
$m = 2 \qquad m = -\frac{1}{2}$

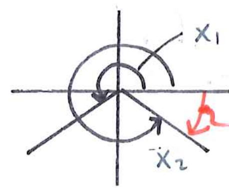
$\sin x = 2 \qquad \sin x = \frac{-1}{2} \qquad \angle x = \sin^{-1}\left(-\frac{1}{2}\right)$

$= -0.524$

No solution

($y = \sin x$ has a max of $\underline{\underline{1}}$)





$X_{ref} = 0.524$

$x_1 = \pi + X_{ref}$

$= \pi + 0.524$

$x_1 = 3.67$

$x_2 = 2\pi - X_{ref}$

$= 2\pi - 0.524$

$x_2 = 5.76$

Practice: 5.4 Worksheet #1abc, 2abc, 3ab