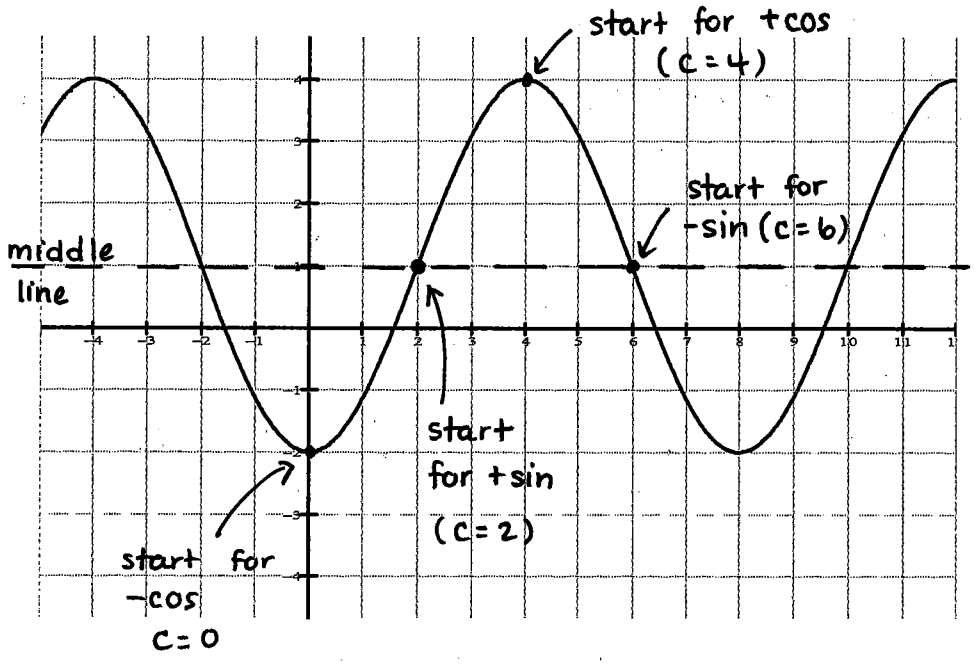
5.4 Equations and Graphs of Trigonometric Functions: Part II

Example 1: The graph below represents a periodic function in the form: $y = AsinB(x - C) + D$ or $y = AcosB(x - C) + D$. Write the equation in both forms.



a) Determine the period

period = 8

$per = \frac{2\pi}{B}$

$8 = \frac{2\pi}{B}$

$B = \frac{2\pi}{8} = \frac{\pi}{4}$

b) Determine amplitude and vertical displacement

$amp = \frac{|max - min|}{2} = \frac{|4-(-2)|}{2} = 3$

So, $A = 3$

vert disp. = max - amp = 4 - 3 = 1

So, $D = 1$

c) Determine the phase shift:

with respect to $y = \sin x$

for + sin , c = 2

for − sin , c = 6

with respect to $y = \cos x$

for + cos , c = 4

for − cos , c = 0

d) Write the equation in the form:

$y = AsinB(x - C) + D$

$y = 3\sin\frac{\pi}{4}(x-2) + 1$

$y = -3\sin\frac{\pi}{4}(x-6) + 1$

$y = AcosB(x - C) + D$

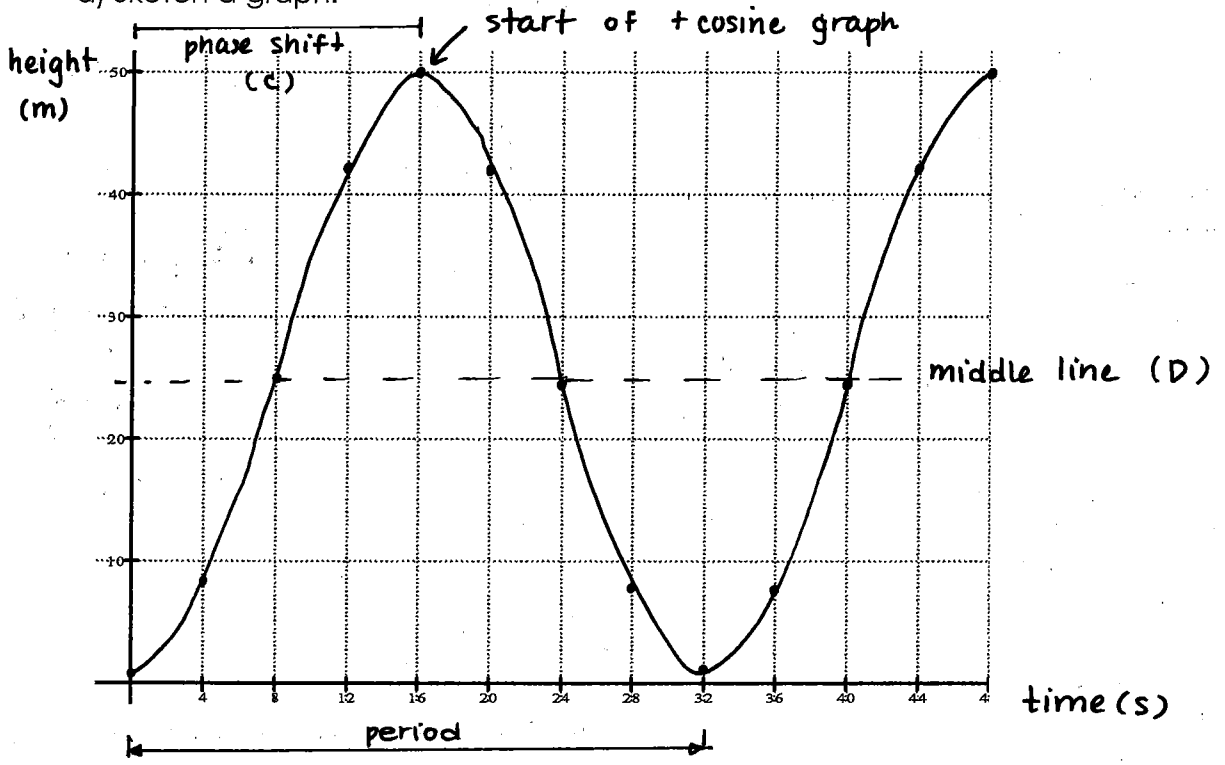
$y = 3\cos\frac{\pi}{4}(x-4) + 1$

$y = -3\cos\frac{\pi}{4}x + 1$

Example 2: The following data describes the height of a seat on a Ferris wheel above the ground, as the wheel rotates:

Time(s)	0	4	8	12	16	20	24	28	32	36	40	44	48
Height (m)	0.8	8.0	25.4	42.8	50	42.8	25.4	8.0	0.8	8.0	25.4	42.8	50

a) Sketch a graph.



b) Determine the period.

period = 32 sec

$per = \frac{2\pi}{B}$

$32B = \frac{2\pi}{B} \cdot B$

$\frac{32B}{32} = \frac{2\pi}{32}$

$B = \frac{\pi}{16}$

c) Determine the amplitude and vertical displacement.

$amp = \frac{|max - min|}{2} = \frac{|50 - 0.8|}{2} = 24.6$

$A = 24.6$

vert. disp. = max − amp

= 50 − 24.6

D = 25.4 (up)

d) Write the equation in the form $y = A\cos B(x - C) + D$

phase shift

C = 16

$y = 24.6 \cos \frac{\pi}{16}(x - 16) + 25.4$

e) How high would you be above the ground after 21 seconds?

↳ y = ?

x = 21

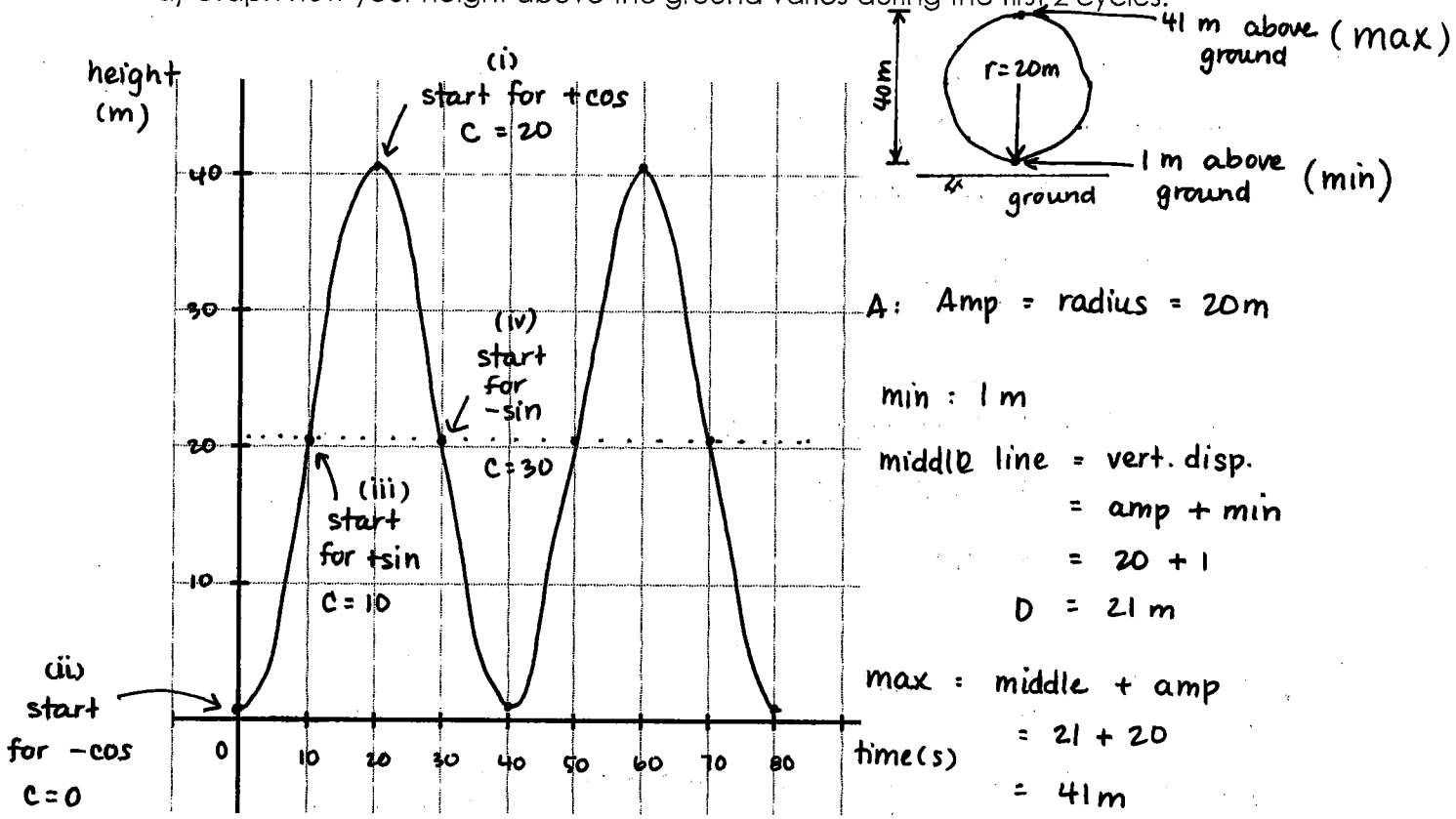
$y = 24.6 \cos \frac{\pi}{16}(21 - 16) + 25.4$

be careful how you input this into your calculator! (calc. must be in radian mode!!)

Example 3: A Ferris wheel has a radius of 20m and rotates once every 40 seconds. Passengers get on 1 m above the ground at point S.

↳ period

a) Graph how your height above the ground varies during the first 2 cycles.



A: Amp = radius = 20m

min : 1 m

middle line = vert. disp.
= amp + min
= 20 + 1
D = 21 m

max : middle + amp
= 21 + 20
= 41 m

per = 40 sec , 2 cycles = 80 sec

b) Write an equation that expresses your height as a function of elapsed time.

$40 = \frac{2\pi}{B}$

$B = \frac{2\pi}{40}$

$B = \frac{\pi}{20}$

$A = 20$

$D = 21$

$C \rightarrow$ varies

(i) $y = A\cos B(x-c) + D$

$y = 20\cos\frac{\pi}{20}(x-20) + 21$

(ii) $y = -20\cos\frac{\pi}{20}x + 21$

(iii) $y = A\sin B(x-c) + D$

$y = 20\sin\frac{\pi}{20}(x-10) + 21$

(iv) $y = -20\sin\frac{\pi}{2}(x-30) + 21$

c) Find your height after 45 seconds.

$y = ?$

$x = 45$

use any of the 4 functions from (b)

$y = 20\cos\left(\frac{\pi}{20}(45-20)\right) + 21$ } calc. in radian mode!

$y = 6.86$ m

Example 4: The height of a tidal wave above mean sea level is related to time by the function: $h(t) = 1.45\cos\frac{2\pi t}{12.4} + 2.35$ where h represents the height, in meters, above mean sea level and t is time, in hours.

a) What is the maximum height of the wave?

max height = vert. disp. + amp

= 2.35 + 1.45

= 3.8m

b) What is the minimum height of the wave?

min height = vert. disp. − amp

= 2.35 − 1.45

= 0.9m

c) What is the period of the wave?

$h(t) = 1.45\cos\frac{2\pi}{12.4}t + 2.35$

$\text{per} = \frac{2\pi}{B} = \frac{2\pi}{\left(\frac{2\pi}{12.4}\right)} = 2\pi \cdot \frac{12.4}{2\pi} =$ 12.4 hours

d) What is the height of the wave 2 hours after high tide?

$t = 2$ hrs

$h(t) = ?$

$h(t) = 1.45\cos\frac{2\pi t}{12.4} + 2.35$

$h(2) = 1.45\cos\frac{2\pi(2)}{12.4} + 2.35$ } radian mode!

$h(2) = 3.12$m.

Practice: p.275 #6, 8ab, 9a-d, 10, 11, 15, 19, 20ab