## 6.1 Rational Expressions

A rational expression is an algebraic fraction with a polynomial in the numerator and/or denominator.

**Examples:** $\frac{1}{2x}$ $\frac{x^2+2}{3}$ $\frac{x+2}{x^2-3x+1}$

**Non-permissible values** are any value(s) of the variable that make the denominator equal to zero.

**Example 1**: Determine the non-permissible values of the following rational expressions:

a) $\frac{2x}{x-2}$

$x-2 \neq 0$

$+2 \quad +2$

$x \neq 2$

b) $\frac{5}{2xy^2}$

$2xy^2 \neq 0$

$x \neq 0$ $\quad y^2 \neq 0$

$y \neq 0$

c) $\frac{5x}{x^2-3x+2}$

$x^2 - 3x + 2 \neq 0$ Factor First!

$-1 \times -2 = 2$

$-1 + -2 = -3$

$(x-1)(x-2) \neq 0$

$x-1 \neq 0$ $\quad x-2 \neq 0$

$x \neq 1$ $\quad x \neq 2$

**Example 2**: Simplify the following rational expressions:

a) $\frac{x+2}{x^2+4x+4}$

① Factor numerator & denominator

$\frac{1(x+2)}{(x+2)(x+2)}$

$x^2+4x+4$

$2 \times 2 = 4$

$2 + 2 = 4$

② state any non-permissible values (n.p.v.)

$(x+2)(x+2) \neq 0$

$x+2 \neq 0$

$x \neq -2$

③ cancel common factors

$\frac{1(x+2)}{(x+2)(x+2)}$

④ Write final answer

$\frac{1}{x+2}$

b) $\frac{m^3 t}{m^2 t^4}$ no factoring necessary

$= m^{3-2} t^{1-4}$ use exponent laws to simplify

$= m^1 t^{-3}$ rewrite w/ positive exponents

$= \frac{m}{t^3}$

n.p.v.

$m^2 t^4 \neq 0$

$m^2 \neq 0$ → $m \neq 0$

$t^4 \neq 0$ → $t \neq 0$

c) $\frac{3x-6}{2x^2+x-10}$ → Factor by decomposition

$-4 \times 5 = -20$

$-4 + 5 = 1$

$2x^2 - 4x + 5x - 10$

$2x(x-2) + 5(x-2)$

$(2x+5)(x-2)$

$= \frac{3(x-2)}{(2x+5)(x-2)}$

$= \frac{3}{2x+5}$

n.p.v.

$(2x+5)(x-2) \neq 0$

$2x+5 \neq 0$ → $x \neq -\frac{5}{2}$

$x-2 \neq 0$ → $x \neq 2$

d) $\frac{6-2m}{m^2-9}$ ← rewrite so variable term is first

← difference of squares

$= \frac{-2m+6}{m^2-9}$ ← factor, 1st term is negative so make "−" part of the GCF

$= \frac{-2(m-3)}{(m+3)(m-3)}$

$= \frac{-2}{m+3}$

n.p.v.

$(m+3)(m-3) \neq 0$

$m+3 \neq 0$ → $m \neq -3$

$m-3 \neq 0$ → $m \neq 3$