

6.3 Proving Identities - part 1

To prove an identity, separately simplify both sides of the identity into identical expressions.

You cannot perform operations across the equal sign. Simplify each side independently.

Example 1: Prove each identity.

$$a) \sec \theta (1 + \cos \theta) = \underbrace{1 + \sec \theta}_{\text{RHS}}$$

$$\sec \theta + \sec \theta \cos \theta = \text{RHS}$$

$$\sec \theta + \frac{1}{\cos \theta} \cdot \cancel{\cos \theta} = \text{RHS}$$

$$\sec \theta + 1 = 1 + \sec \theta$$

or Rewrite $\sec \theta$ first

$$\frac{1}{\cos \theta} (1 + \cos \theta) = \text{RHS}$$

$$\frac{1}{\cos \theta} + 1 = \text{RHS}$$

$$\sec \theta + 1 = 1 + \sec \theta$$

b) $\tan \theta + \cot \theta = \sec \theta \csc \theta$ Rewrite the more complicated side

$$\frac{\sin \theta \cdot \sin \theta}{\sin \theta \cos \theta} + \frac{\cos \theta \cdot \cos \theta}{\sin \theta \cos \theta} = \text{RHS}$$

Find a common denominator

$$\frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta \cos \theta} = \text{RHS} \quad \leftarrow \text{Pythagorean Identity}$$

$$\frac{1}{\sin \theta \cos \theta} = \text{RHS}$$

$$\frac{1}{\sin \theta} \cdot \frac{1}{\cos \theta} = \text{RHS}$$

$$\csc \theta \sec \theta = \text{RHS}$$

$$c) \underbrace{\sec^4 x - \sec^2 x}_{\text{Factor out a GCF}} = \underbrace{\tan^4 x + \tan^2 x}_{\text{RHS}}$$

Factor out a GCF

$$\sec^2 x (\sec^2 x - 1) = \text{RHS}$$

Pythagorean Identity

$$1 + \tan^2 x = \sec^2 x \rightarrow \tan^2 x = \sec^2 x - 1$$

$$(1 + \tan^2 x)(\tan^2 x) = \text{RHS}$$

$$\tan^2 x + \tan^4 x = \text{RHS}$$

$$d) \frac{1}{1+\cos\theta} + \frac{1}{1-\cos\theta} = 2\csc^2\theta$$

Work on the more complicated side (left)

Find a common denominator

$$\frac{1}{(1+\cos\theta)(1-\cos\theta)} + \frac{1}{(1-\cos\theta)(1+\cos\theta)} = \text{RHS}$$

$$\frac{1 - \cancel{\cos\theta} + (1 + \cancel{\cos\theta})}{(1+\cos\theta)(1-\cos\theta)} = \text{RHS}$$

$$\frac{2}{1 - \cancel{\cos\theta} + \cancel{\cos\theta} - \cos^2\theta} = \text{RHS}$$

Pythagorean Identity \rightarrow $\frac{2}{1 - \cos^2\theta} = \text{RHS}$

$$\frac{2}{\sin^2\theta} = \text{RHS}$$

$$2\csc^2\theta = \text{RHS}$$