6.3 Proving Identities – part 2

Example 1: Expand using double angle identities

a) $\sin 10\theta$

if $\sin 2\theta = 2\sin\theta\cos\theta$

$\sin 10\theta = \sin 2(5\theta)$

$= 2\sin 5\theta \cos 5\theta$

b) $\sin 6x$

$\sin 6x = \sin 2(3x) = 2\sin 3x \cos 3x$

c) $\cos 8\theta$

$\cos 8\theta = \cos 2(4\theta)$

$= \cos^2 4\theta - \sin^2 4\theta$

d) $\tan 4\theta$

$\tan 2(2\theta) = \dfrac{2\tan 2\theta}{1 - \tan^2 2\theta}$

Example 2: Prove each identity.

a) $\tan 2\theta = \frac{\sin 4\theta}{1+\cos 4\theta}$

$\sin 4\theta = \sin 2(2\theta) = 2\sin 2\theta \cos 2\theta$

$\cos 4\theta = \cos 2(2\theta) = 2\cos^2 2\theta - 1$

$LHS = \dfrac{2\sin 2\theta \cos 2\theta}{\cancel{1} + (2\cos^2 2\theta \cancel{- 1})}$

$LHS = \dfrac{\cancel{2}\sin 2\theta \cancel{\cos 2\theta}}{\cancel{2}\cos^{\cancel{2}} 2\theta}$

$LHS = \dfrac{\sin 2\theta}{\cos 2\theta}$

$LHS = \tan 2\theta$

b) $\cos 2\theta = \frac{\csc^2\theta - 2}{\csc^2\theta}$

$$LHS = \frac{\csc^2\theta}{\csc^2\theta} - \frac{2}{\csc^2\theta}$$

$$LHS = 1 - \frac{2}{\csc^2\theta}$$

$$LHS = 1 - 2 \cdot \frac{1}{\csc^2\theta}$$

$$LHS = 1 - 2\sin^2\theta$$

$$LHS = \cos 2\theta$$

Example 3: Simplify

a) $\frac{3}{7}+\frac{2}{5}$

$$= \frac{3 \cdot 5}{7 \cdot 5} + \frac{2 \cdot 7}{5 \cdot 7}$$

$$= \frac{15}{35} + \frac{14}{35}$$

$$= \frac{15 + 14}{35}$$

$$= \frac{29}{35}$$

b) $\frac{\sqrt{3}}{(5-\sqrt{2})} \cdot \frac{(5+\sqrt{2})}{(5+\sqrt{2})}$

rationalize the denominator (with 2 terms)
→ multiply numerator and denominator by the conjugate of the denominator

$$= \frac{5\sqrt{3} + \sqrt{6}}{25 + \cancel{5\sqrt{2}} - \cancel{5\sqrt{2}} - \sqrt{4}}$$

$$= \frac{5\sqrt{3} + \sqrt{6}}{25 - 2}$$

$$= \frac{5\sqrt{3} + \sqrt{6}}{23}$$

Example 4: Prove each identity.

a) $\underbrace{\dfrac{\cos x}{1-\sin x}}$ $= \dfrac{1+\sin x}{\cos x}$

more complicated denominator (has 2 terms)

Multiply by the conjugate

$$\frac{\cos x}{(1-\sin x)}\frac{(1+\sin x)}{(1+\sin x)} = \text{RHS}$$

$$\frac{\cos x(1+\sin x)}{1+\cancel{\sin x}-\cancel{\sin x}-\sin^2 x} = \text{RHS}$$

$$\frac{\cos x(1+\sin x)}{1-\sin^2 x} = \text{RHS}$$

Pythagorean Identity

$$\frac{\cancel{\cos x}(1+\sin x)}{\cos^{\cancel{2}} x} = \text{RHS}$$

$$\frac{1+\sin x}{\cos x} = \text{RHS}$$

b' $\dfrac{\sin\theta+\tan\theta}{1+\cos\theta} = \tan\theta$ rewrite

$$\frac{\sin\theta + \frac{\sin\theta}{\cos\theta}}{1+\cos\theta} = RHS$$

Find a common denominator

$$\frac{\frac{\sin\theta}{1}\cdot\frac{\cos\theta}{\cos\theta} + \frac{\sin\theta}{\cos\theta}}{1+\cos\theta} = RHS$$

$$\frac{\frac{\sin\theta\cos\theta + \sin\theta}{\cos\theta}}{1+\cos\theta} = RHS$$

factor out $\sin\theta$

$$\frac{\frac{\sin\theta(\cos\theta+1)}{\cos\theta}}{1+\cos\theta} = RHS$$

← rewrite as a multiplication

$$\frac{\sin\theta\,\cancel{(\cos\theta+1)}}{\cos\theta}\cdot\frac{1}{\cancel{(1+\cos\theta)}} = RHS$$

$$\frac{\sin\theta}{\cos\theta} = RHS$$

$$\tan\theta = RHS$$

Practice: p.314 #7, 10 – 13, 15b, 18