

6.4 Solving Trig Equations Using Identities – part 2

General Solutions

Example 1: Solve $\sin 2\theta = \sqrt{2} \cos \theta$. Give the general solution in radians.

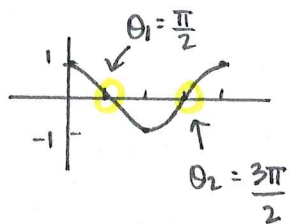
rewrite

$$2\sin\theta \cos\theta = \sqrt{2} \cos\theta$$

$$2\sin\theta \cos\theta - \sqrt{2} \cos\theta = 0$$

$$\cos\theta (2\sin\theta - \sqrt{2}) = 0$$

$$\cos\theta = 0$$

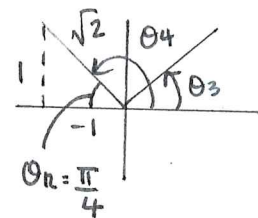


$$2\sin\theta - \sqrt{2} = 0$$

$$\sin\theta = \frac{\sqrt{2} \cdot \sqrt{2}}{2 \cdot \sqrt{2}}$$

$$= \frac{2}{2\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}}$$



$$\theta_3 = \theta_R = \frac{\pi}{4}$$

$$\theta_4 = \pi - \theta_R$$

$$= \pi - \frac{\pi}{4}$$

$$\theta_4 = \frac{3\pi}{4}$$

General solution = solution + period (n), $n \in \mathbb{I}$

$$\theta = \frac{\pi}{2} + 2\pi n$$

$$\theta = \frac{3\pi}{2} + 2\pi n$$

$$\theta = \frac{\pi}{4} + 2\pi n$$

$$\theta = \frac{3\pi}{4} + 2\pi n$$

where $n \in \mathbb{I}$

since these answers are only separated by π , we could combine them and write $\theta = \frac{\pi}{2} + \pi n$

Example 2: Solve $2 \sin \theta = 7 - 3 \csc \theta$. Give the general solution in radians.

↑ rewrite

$$2 \sin \theta = 7 - 3 \left(\frac{1}{\sin \theta} \right)$$

$$2 \sin \theta = 7 - \frac{3}{\sin \theta} \quad \begin{matrix} (\sin \theta) & (\sin \theta) \\ \uparrow & \uparrow \end{matrix}$$

$$2 \sin^2 \theta = 7 \sin \theta - 3$$

$$2 \sin^2 \theta - 7 \sin \theta + 3 = 0 \quad \left. \vphantom{2 \sin^2 \theta - 7 \sin \theta + 3 = 0} \right\} \text{factor using decomposition}$$

$$(\sin \theta - 3)(2 \sin \theta - 1) = 0$$

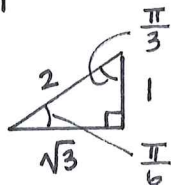
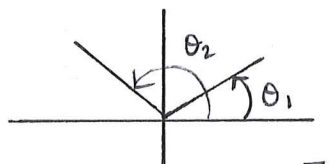
$$\downarrow$$

$$\sin \theta = 3$$

No solution

$$\downarrow$$

$$\sin \theta = \frac{1}{2}$$



$$\theta_R = \frac{\pi}{6}$$

$$\theta_1 = \theta_R$$

$$\theta_1 = \frac{\pi}{6}$$

$$\theta_2 = \pi - \theta_R$$

$$= \frac{6\pi}{6} - \frac{\pi}{6}$$

$$\theta_2 = \frac{5\pi}{6}$$

$$2m^2 - 7m + 3 = 0$$

$$\begin{matrix} \uparrow & & \uparrow \\ (2)(3) = 6 & & \frac{-1 \times -6 = 6 \\ -1 + -6 = -7 \end{matrix}$$

$$2m^2 - m - 6m + 3 = 0$$

$$m(2m-1) - 3(2m-1) = 0$$

$$(m-3)(2m-1) = 0$$

General solution :

$$\theta = \frac{\pi}{6} + 2\pi n$$

$$\theta = \frac{5\pi}{6} + 2\pi n$$

where $n \in \mathbb{I}$