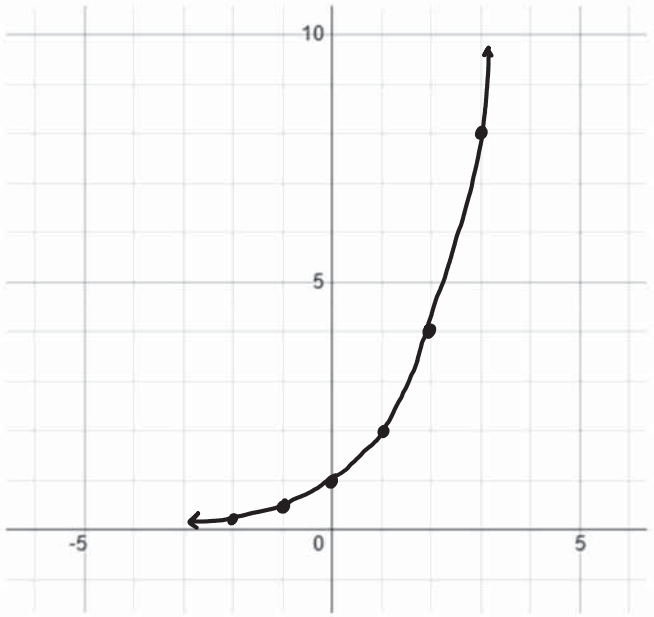
7.1 Characteristics of Exponential Functions

An exponential function is of the form: $y = c^x$ where

c is a constant and $c > 0$; x is a variable

Example 1: Use a table of values to sketch the graph of $y = 2^x$ on the grid below.

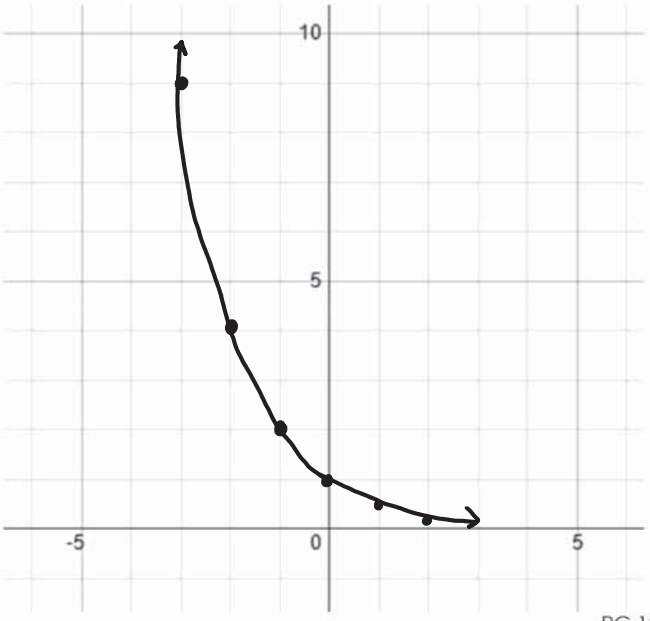
x	y	
-2	$\frac{1}{4}$	$\leftarrow y = 2^{-2} = \frac{1}{2^2}$
-1	$\frac{1}{2}$	$\leftarrow y = 2^{-1}$
0	1	$y = 2^0$
1	2	$y = 2^1$
2	4	$y = 2^2$
3	8	$y = 2^3$



The x-axis is a horizontal asymptote.

Example 2: Use a table of values to sketch the graph of $y = \left(\frac{1}{2}\right)^x$ on the grid below.

x	y	
-3	8	$y = (\frac{1}{2})^{-3} = (\frac{2}{1})^3$
-2	4	$y = (\frac{1}{2})^{-2} = (\frac{2}{1})^2$
-1	2	$y = (\frac{1}{2})^{-1} = (\frac{2}{1})^1$
0	1	$y = (\frac{1}{2})^0$
1	$\frac{1}{2}$	$y = (\frac{1}{2})^1$
2	$\frac{1}{4}$	$y = (\frac{1}{2})^2$



$y = 2^x$ and $y = (\frac{1}{2})^x$ are reflections of each other over the y-axis.

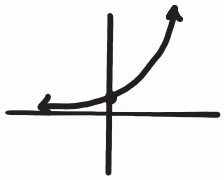
The graph of an exponential function $y = c^x$ is:

- increasing for $c > 1$

ex: $y = 2^x$

$y = 7.3^x$

$y = \left(\frac{11}{4}\right)^x$

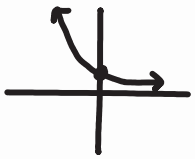


- decreasing for $0 < c < 1$

ex: $y = \left(\frac{1}{3}\right)^x$

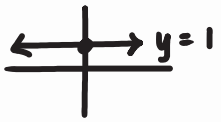
$y = 0.2^x$

$y = \left(\frac{5}{11}\right)^x$

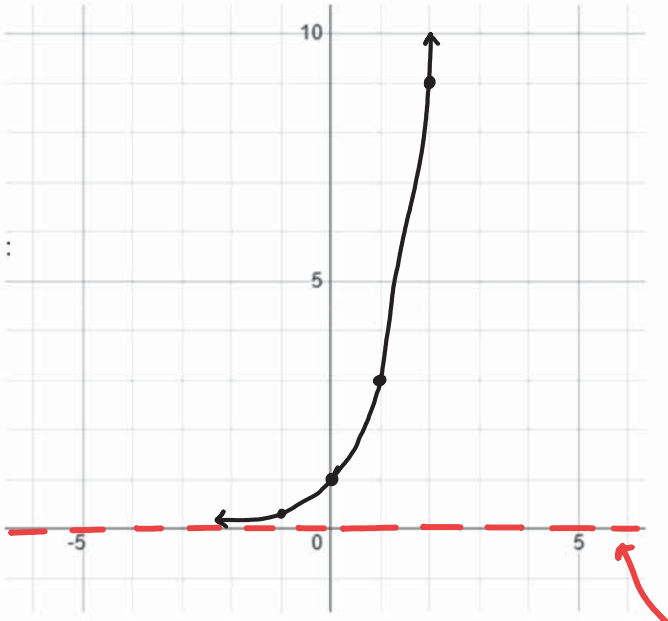


- constant for $c = 1$

$y = 1^x$



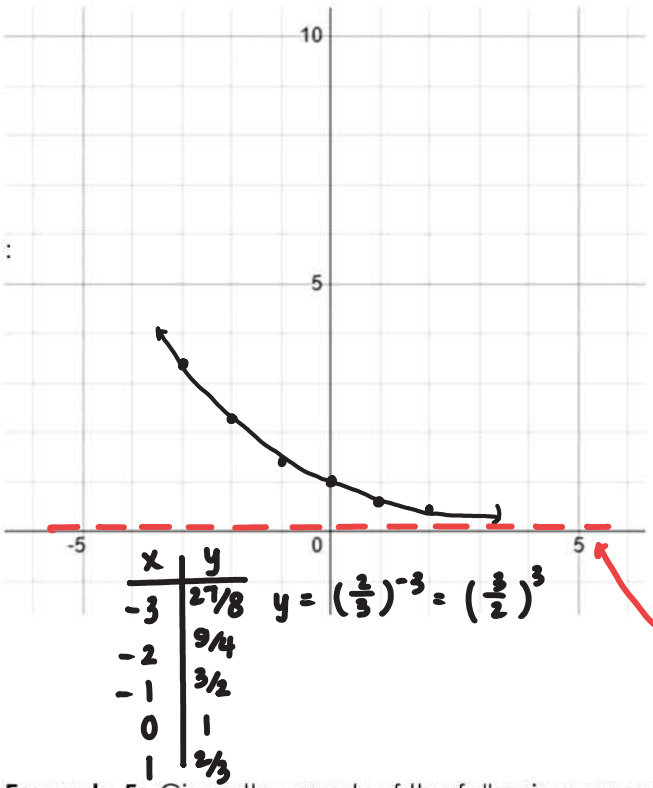
Example 3: Graph the function $y = 3^x$ and state the following:



x	y
-1	1/3
0	1
1	3
2	9

Domain	$\{x \mid x \in \mathbb{R}\}$
Range	$\{y \mid y > 0, y \in \mathbb{R}\}$
Increasing or Decreasing	increasing ($c > 1$)
x-intercept	none
y-intercept	$(0, 1)$
Equation of the horizontal asymptote	$y = 0$

Example 4: Graph the function $y = \left(\frac{2}{3}\right)^x$ and state the following:

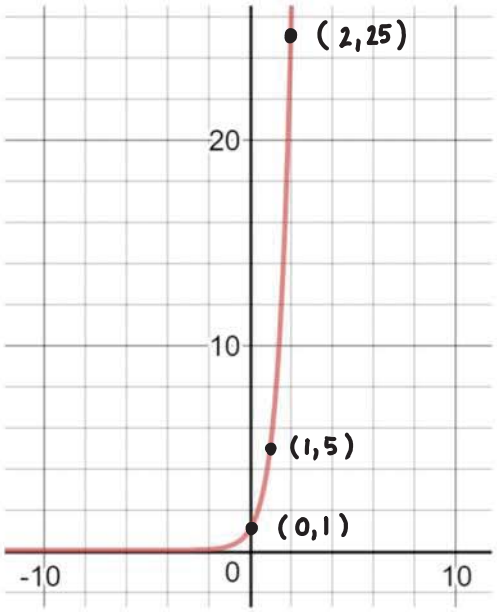


x	y
-3	27/8
-2	9/4
-1	3/2
0	1
1	2/3

$y = \left(\frac{2}{3}\right)^{-3} = \left(\frac{3}{2}\right)^3$

Domain	$\{x \mid x \in \mathbb{R}\}$
Range	$\{y \mid y > 0, y \in \mathbb{R}\}$
Increasing or Decreasing	decreasing ($c < 1$)
x-intercept	none
y-intercept	(0,1)
Equation of the horizontal asymptote	$y = 0$

Example 5: Given the graph of the following exponential function state the equation.



↳ $y = c^x$

increasing so $c > 1$

use pt. to find c

(1,5) : $5 = c^1$, → $c = 5$

x y

or

(2,25) : $25 = c^2$

So... $y = 5^x$

Example 6: A certain bacteria culture triples every week.

a) Write an exponential function that models this situation.

rate of change = triples → $c = 3$

Initial # of bacteria = A_0

x = time in weeks

$y = A_0 \cdot c^x$ → $y_0 = A_0(3)^x$

b) If there is only one bacteria present initially, how many bacteria are there after 11 weeks?

$A_0 = 1$ → $x = 11$

$y = A_0(3)^x$

$y = 1 \cdot 3^{11}$

$y = 1(177147)$

There are 177,147 bacteria after 11 weeks.

Example 7: A new car depreciates by 12% each year. Write an exponential function that models the value of the car, V, as a function of time.

decreases by 12% (0.12)

rate of change = $1 - 0.12$

$c = 0.88$

$V = V_0 \cdot c^t$

$V = V_0(0.88)^t$

where V_0 = initial value of the car

t = time in years

Example 8: The population of a town is growing by 6% each year. Write an exponential function that models the population, P, as a function of time. What assumptions did you make?

increases by 6% (0.06)

rate of change = $1 + 0.06$

$c = 1.06$

$P = P_0 \cdot c^t$

$P = P_0(1.06)^t$

P_0 = initial population

t = time in years

Practice: p.342 #1 – 7, 9

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