7.2 Transformations of Exponential Functions

$$y = a(c)^{b(x-h)} + k$$

- a vert. stretch by a factor of $|a|$; if $a<0$ → reflection over x-axis
- b horiz. stretch by a factor of $\left|\frac{1}{b}\right|$; if $b<0$ → reflection over y-axis
- h horiz. translation ↔
- k vert. translation ↕ (locates the horizontal asymptote)

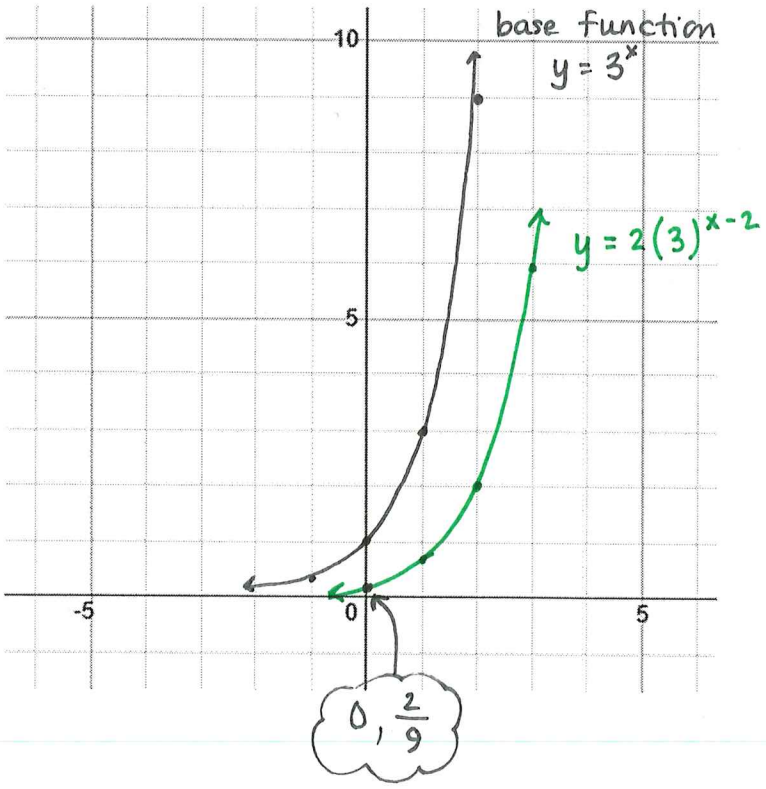
Example 1: Graph the base function $y = 3^x$ and the transformed function $y = 2(3)^{x-2}$ on the same grid. Describe the transformations.

Transformations:

$a = 2$ vert. stretch by factor of 2

$h = 2$ horiz. translation 2 units right

+2	x	y · 2
1	~~-1~~	~~1/3~~ 2/3
2	~~0~~	~~1~~ 2
3	~~1~~	~~3~~ 6
4	~~2~~	~~9~~ 18



Find the intercepts of $y = 2(3)^{x-2}$

x-intercepts

none - x-axis is an asymptote

y-intercept

occurs when $x = 0$

$y = 2(3)^{0-2}$

$y = 2(3)^{-2}$

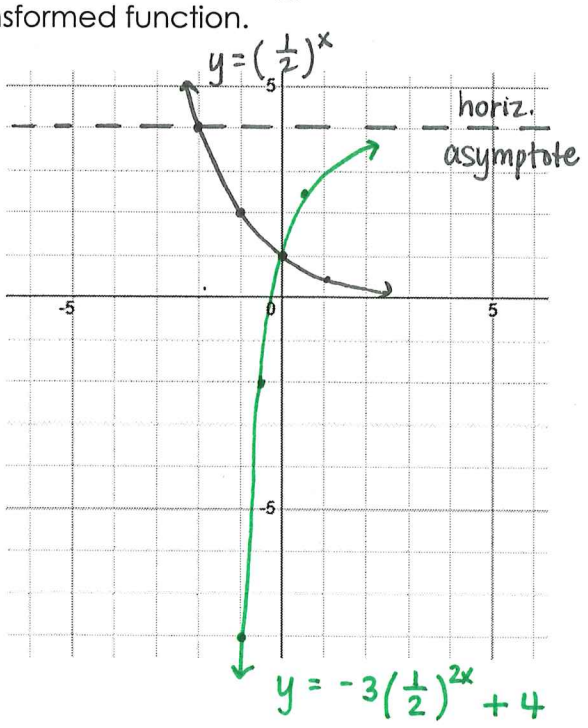
$= 2\left(\frac{1}{9}\right)$

$y = \frac{2}{9}$ or $\left(0, \frac{2}{9}\right)$

Example 2: Graph the base function $y = \left(\frac{1}{2}\right)^x$ and the transformed function $y = -3\left(\frac{1}{2}\right)^{2x} + 4$ on the same grid. State the asymptotes, domain and range of the transformed function.

a b k

$\frac{1}{2} \cdot x$		$y \cdot -3$		$+4$
-1	~~-2~~	~~4~~	~~-12~~	-8
$-\frac{1}{2}$	~~-1~~	~~2~~	~~-6~~	-2
0	~~0~~	~~1~~	~~-3~~	1
$\frac{1}{2}$	~~1~~	~~$\frac{1}{2}$~~	~~$-\frac{3}{2}$~~	$\frac{5}{2}$



Asymptote: $y = 4$

Domain: $\{x \mid x \in \mathbb{R}\}$

Range: $\{y \mid y < 4;\ y \in \mathbb{R}\}$

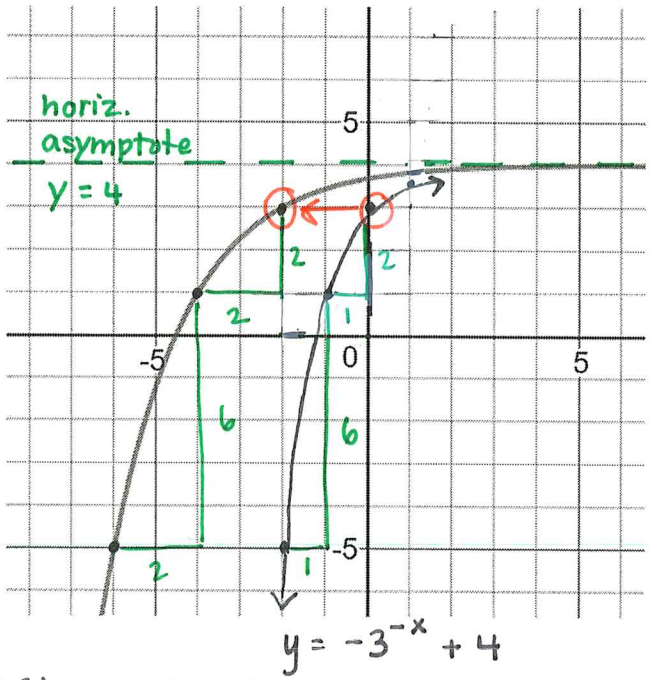
Example 3: The base equation is $y = 3^x$. Write an equation for the transformed function.

$$y = a(3)^{b(x-h)} + k$$

- horiz. asymptote $y = 4$
  so $K = 4$
- reflection over x & y axes
  so "a" and "b" are negative
- use an intermediate graph to figure out other transformations.

$$y = -3^{-x} + 4$$

$-1 \cdot x$		$y \cdot -1$		$+4$
1	~~-1~~	$\frac{1}{3}$	$-\frac{1}{3}$	$\frac{11}{3}$
0	~~0~~	~~1~~	~~-1~~	3
-1	~~1~~	~~3~~	~~-3~~	1
-2	~~2~~	~~9~~	~~-9~~	-5



Use y-int $(0,3) \longrightarrow (-2,3)$
horizontal translation 2 units left, $h = -2$

Use Δ to find stretches
- no change in vert. ($a = 1$)
- horiz. stretch by 2, so $b = \frac{1}{2}$

$$y = -3^{-\frac{1}{2}(x+2)} + 4$$

Example 4: A scientist finds that when you consume caffeine, it takes about 4 hours for the amount of caffeine to drop by 40%.

decreased by 40% (0.40)

a) Write an equation to represent the percentage, P, of the caffeine left in the body after time t.

Where $P = a(c)^{\frac{t}{T}}$.

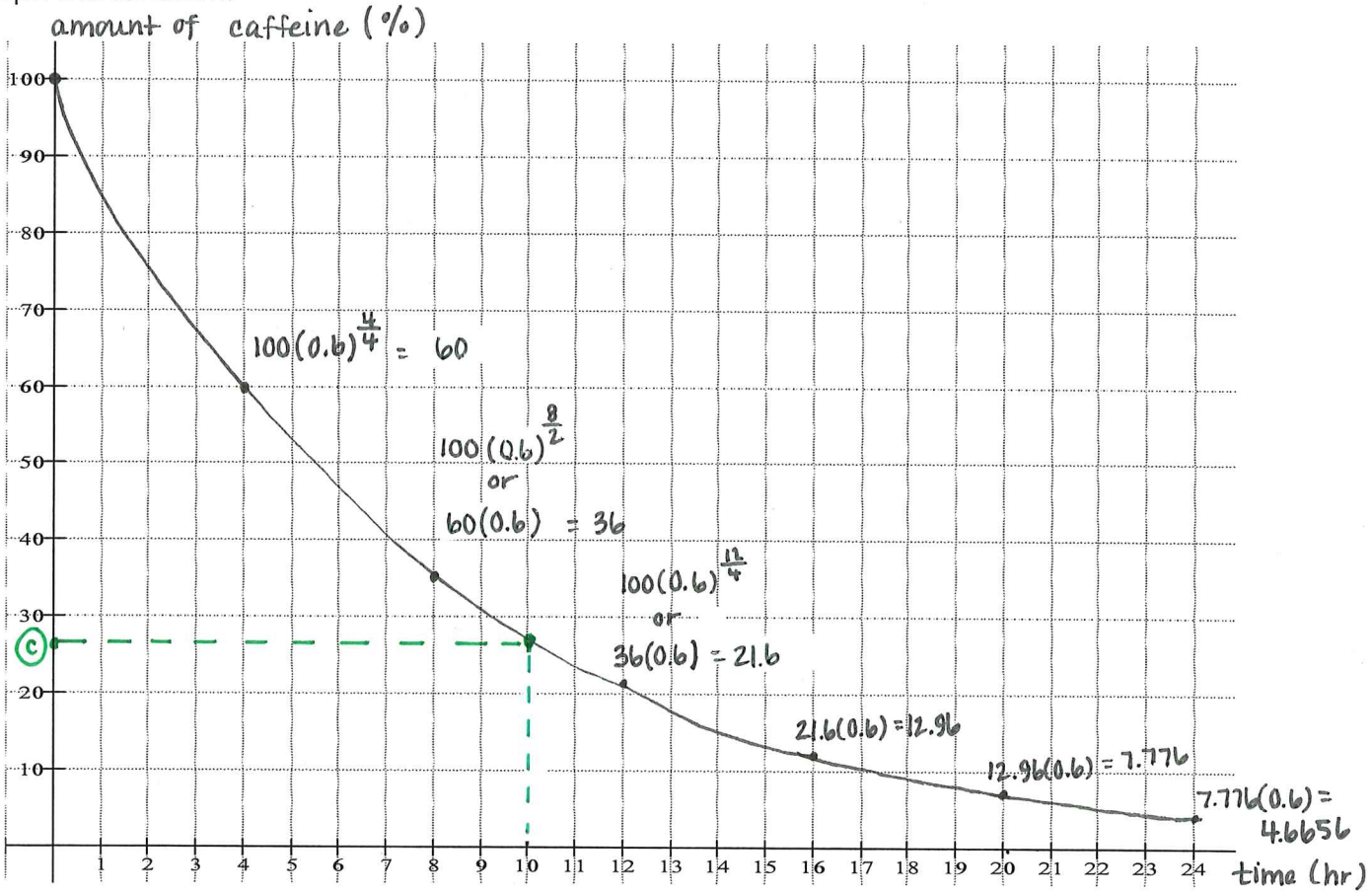
t = ellapsed time (variable)

T = time interval (can't control)

initial amount of caffeine: 100% or 1 or 100

% decrease: $c = 1 - 0.40$, $c = 0.60$

b) Graph the function.



c) Using the graph estimate the percent of caffeine that remains after 10 hours.

approx. 27%

d) Calculate the percent of caffeine that remains after 10 hours using the equation from (a).

$t = 10$ hrs

$P = 100(0.6)^{\frac{t}{4}}$

$= 100(0.6)^{\frac{10}{4}}$

$P = 100(0.6)^{2.5}$

evaluate this next

$P = 100(0.27885)$

$P = 27.89\%$

Practice: p.354 #1, 2, 3efg, 4, 6, 7bd, 8bd, 9