7.3 Solving Exponential Equations – part 2

A. Growth / Decay

$$A = A_o(c)^{\frac{t}{T}}$$

$A =$ final amount

$A_o =$ intial amount

$c =$ rate of change

if $c > 0 \rightarrow$ growth

if $0 < c < 1 \rightarrow$ decay

$t =$ ellapsed time

$T =$ interval time (how often the rate changes)

Example 1: A bacteria culture starts with 6250 bacteria. After 4 hours, the count is 50,000. How often does the bacteria double?

$A_o = 6250$

$A = 50{,}000$

$c = 2$ (doubling)

$t = 4$ hrs

$T = ?$

$$A = A_o(c)^{\frac{t}{T}}$$

$$\frac{50000}{6250} = \frac{6250(2)^{\frac{4}{T}}}{6250}$$

$$8 = 2^{\frac{4}{T}}$$

rewrite

$$2^3 = 2^{\frac{4}{T}}$$

exp. only : $T \cdot 3 = \frac{4}{\cancel{T}} \cdot \cancel{T}$

$T = \frac{4}{3}$ hrs or $4\frac{1}{3}$ hrs or 4 hr 20 min

Example 2: A radioactive substance has a half – life of 18.2 hours. How long will it take until only 12.5 % of the sample remains?

$A_o = 100\%$

$A = 12.5\%$

$c = \frac{1}{2}$ (half-life)

$t = ?$

$T = 18.2$ hrs

$$A = A_o(c)^{\frac{t}{T}}$$

$$\frac{12.5}{100} = \frac{\cancel{100}\left(\frac{1}{2}\right)^{\frac{t}{18.2}}}{\cancel{100}}$$

$$0.125 = \left(\frac{1}{2}\right)^{\frac{t}{18.2}}$$

$$\left(\frac{1}{2}\right)^3 = \left(\frac{1}{2}\right)^{\frac{t}{18.2}}$$

exp. only : $18.2 \cdot 3 = \frac{t}{\cancel{18.2}} \cdot \cancel{18.2}$

54.6 hours = t

Example 3: A painting's value doubles every 5 years. It is currently worth $1000. How much time is needed for it to be worth $32,000?

$A_0 = 1000$

$A = 32000$

$c = 2$

$t = ?$

$T = 5$

$$A = A_0(c)^{\frac{t}{T}}$$

$$\frac{32000}{1000} = \frac{1000(2)^{\frac{t}{5}}}{1000}$$

$$32 = 2^{\frac{t}{5}}$$

$$2^5 = 2^{\frac{t}{5}}$$

exp. only $:5 \cdot 5 = \frac{t}{\cancel{5}} \cdot \cancel{5}$

25 years $= t$

B. Compound Interest

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$A =$ Final amount

$P =$ principle (initial amount)

$r =$ interest rate (decimal form)

$t =$ time in years

$n =$ # of compounding periods per year

Compounding Periods:

Annually (yearly)	$n = 1$	Monthly	$n = 12$
Semi – Annually	$n = 2$	Weekly	$n = 52$
Quarterly	$n = 4$	Daily	$n = 365$

Example 4: You invest $250 at 3% compounded semi – annually. How much will it be worth in twelve years?

$A = ?$

$P = 250$

$r = 3\% = 0.03$

$n = 2$

$t = 12$

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$= 250\left(1 + \frac{0.03}{2}\right)^{2 \cdot 12}$$

$$= 250\underbrace{(1.015)^{24}}_{\text{evaluate this next}}$$

$$= 250(1.42950\ldots)$$

$$= 357.3757$$

The investment is worth $357.38 in 12 years.

Practice: p.364 #11ab, 13ab, Worksheet 7.3