

7.3 Models of Growth and Decay Exponential Functions

Write an exponential function for each situation then solve the problem.

1. There are now 300 insects in a colony. The population doubles every 5 days. What is the population in 18 days?

$$A_0 = 300$$

$$t = 18$$

$$c = 2 \quad T = 5$$

$$A = A_0(C)^{\frac{t}{T}}$$

$$= (300)(2)^{\frac{18}{5}}$$

$$= (300)(2)^{3.6}$$

$$= 3637.72$$

round down

$$A = 3637 \text{ bugs}$$

2. For every meter a diver descends below the surface, the light intensity is reduced by 2.5%. P is the percent of surface light present. At a depth of 10m how much light remains?

$$A_0 = 100\%$$

$$A = ?$$

$$A = A_0(C)^n$$

$$A = (100)(0.975)^{10}$$

$$A = 77.6\%$$

$$n = 10$$

$$c = 1 - 0.025$$

$$= 0.975$$

3. A radioactive substance has a half-life of 6 years. If 20 grams are present initially, how much will remain after 2 years?

$$t = 2$$

$$c = \frac{1}{2}$$

$$T = 6$$

$$A_0 = 20$$

$$A = ?$$

$$A = (20)\left(\frac{1}{2}\right)^{\frac{2}{6}}$$

$$= 20(0.7937)$$

$$A = 15.87g$$

Write an exponential function for each situation then solve the problem algebraically.

4. The half-life of radioactive iodine is 8.2 days. After how long will only 25% of the iodine be present?

$$c = \frac{1}{2}$$

$$T = 8.2$$

$$t = ?$$

$$A = 25, A_0 = 100$$

$$\frac{25}{100} = \frac{100}{100} \left(\frac{1}{2}\right)^{\frac{t}{8.2}}$$

$$0.25 = (0.5)^{\frac{t}{8.2}}$$

$$(0.5)^2 = (0.5)^{\frac{t}{8.2}}$$

$$2 = \frac{t}{8.2}$$

$$t = 16.4 \text{ days.}$$

5. A bacteria starts with 6250 bacteria and doubles every 3 hours. When will the bacteria count be 50000?

$$A$$

$$A_0$$

$$c = 2$$

$$T = 3$$

$$t = ?$$

$$\frac{50000}{6250} = \frac{6250}{6250} (2)^{\frac{t}{3}}$$

$$8 = 2^{\frac{t}{3}}$$

$$2^3 = 2^{\frac{t}{3}}$$

Exp only

$$3 = \frac{t}{3}$$

$$t = 9 \text{ hours}$$

6. A colony of insects numbers $\frac{500}{A_0}$ and doubles every $\frac{8}{T}$ days. How long ago was the population $\frac{125}{A}$? $C=2$ $t=?$

A

$$\frac{125}{500} = \frac{500}{500} (2)^{\frac{t}{8}}$$

$$0.25 = 2^{\frac{t}{8}}$$

$$2^{-2} = 2^{\frac{t}{8}}$$

Exp. only $-2 = \frac{t}{8}$

-16 days = t
16 days ago

7. A radioactive substance has a half-life of $\frac{3.5}{T}$ years. How long will it take for only $\frac{6.25}{A}$ of it to remain? $A_0 = 100$ $t=?$

$$\frac{6.25}{100} = \frac{100}{100} \left(\frac{1}{2}\right)^{\frac{t}{3.5}}$$

$$0.0625 = (0.5)^{\frac{t}{3.5}}$$

$$(0.5)^4 = (0.5)^{\frac{t}{3.5}}$$

Exp. only $4 = \frac{t}{3.5}$

$t = 14$ years

8. A painting triples in value every $\frac{8}{T}$ years. It is currently worth \$1000. When will the painting be worth \$243000? $A_0 = 1000$ $t=?$

A

$$\frac{243000}{1000} = \frac{1000}{1000} (3)^{\frac{t}{8}}$$

$$243 = (3)^{\frac{t}{8}}$$

$$3^5 = 3^{\frac{t}{8}}$$

Exp. only $5 = \frac{t}{8}$

40 years = t

9. A piece of machinery valued at \$30,000 depreciates at a rate of 10% per year. How much will it be worth in 7 years? $A_0 = 30,000$ $C = 1 - 0.1 = 0.9$ $t = 7$ $A = ?$

$t = 7$

$$A = 30,000 (0.9)^7$$

$$= 30,000 (0.4783)$$

$A = \$14348.91$

10. \$1000 is invested at a rate of 3.2% compounded monthly. How much will it be worth in 50 years? $P = 1000$ $r = 0.032$ $n = 12$ $A = ?$ $t = 50$

P

$$A = P \left(1 + \frac{r}{n}\right)^{nt}$$

$$= 1000 \left(1 + \frac{0.032}{12}\right)^{12 \cdot 50}$$

$$= 1000 (1.0027)^{600}$$

$A = \$4942.50$