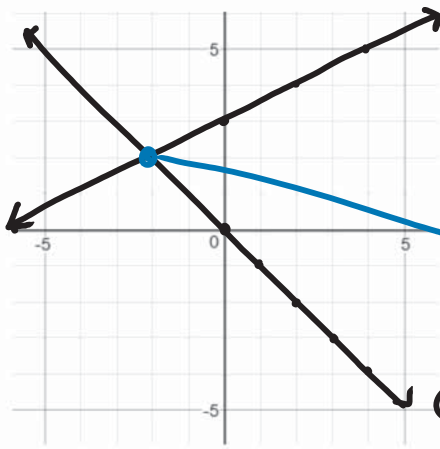
7.6 Properties of Linear Systems

When we graph a system of linear equations, the solution is the intersection point of the two equations.

Example 1: Determine the number of solutions of each linear system.

1) Solve graphically : ① $y = -x$ and ② $2y = x + 6$



① $y = -x$ $\quad (m = -1,\ b = 0)$

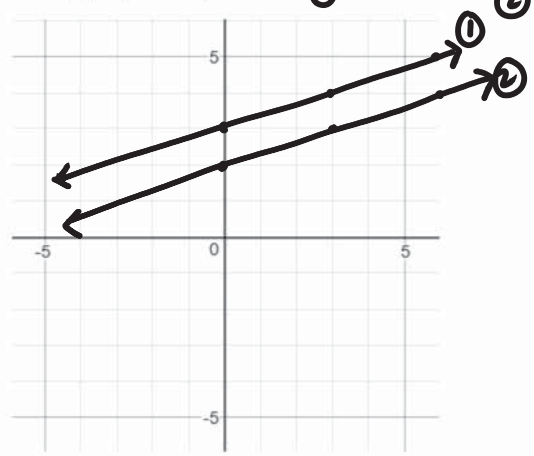
② $\frac{2y}{2} = \frac{x}{2} + \frac{6}{2}$

$y = \frac{1}{2}x + 3$ $\quad (m = \frac{1}{2},\ b = 3)$

Because these graphs have different slopes, they intersect at exactly one point.

Solution: There is one solution

2) Solve graphically : ① $y = \frac{1}{3}x + 3$ and ② $x - 3y + 6 = 0$



① $y = \frac{1}{3}x + 3$ $\quad (m = \frac{1}{3},\ b = 3)$

② $x - 3y + 6 = 0$

$-x \quad -6 \quad -x - 6$

$\frac{-3y}{-3} = \frac{-x}{-3} \frac{-6}{-3}$

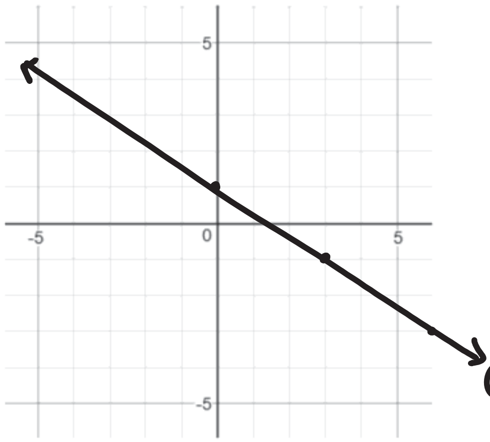
$y = \frac{1}{3}x + 2$ $\quad (m = \frac{1}{3},\ b = 2)$

Because these graphs have the same slope and different y-intercepts, they are parallel.

Solution : There is no solution

(parallel lines never intersect)

3) Solve graphically : ① $y=-\frac{2}{3}x+1$ and ② $2x+3y-3=0$



① and ②

① $y=-\frac{2}{3}x+1$ $\left(m=\frac{-2}{3},\ b=1\right)$

② $2x+3y-3=0$

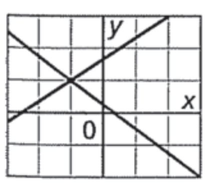
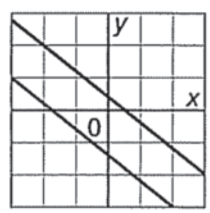
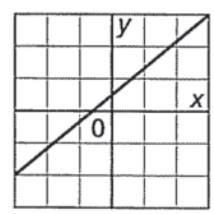
$-2x \quad +3 \quad -2x+3$

$\frac{3y}{3}=\frac{-2x}{3}+\frac{3}{3}$

$y=-\frac{2}{3}x+1$

Because these graphs have the same slope and same y-intercept, they are coincident.

Solution: There an infinite number of solutions.

Possible Solutions for a Linear System		
Intersecting Lines	**Parallel Lines**	**Coincident Lines**
One Solution	No Solution	Infinite Solutions
		

Example 2: Determine the number of solutions without solving (examine the slopes and y-intercepts).

a) $y=\frac{2}{3}x+1$ and $y=\frac{2}{3}x-4$

same slope

different y-intercepts

these lines are parallel

therefore there is no solution

c) $y=-3x-6$ and $2y=-6x-12$

rewrite first

$\frac{2y}{2}=\frac{-6x}{2}-\frac{12}{2}$

$y=-3x-6$

same slope, same y-intercept, these lines are coincident therefore there is an infinite number of solutions.

Practice: p. 448 # 4, 6, 7, 10