

## 9.2 Quadratic Inequalities in One Variable

A quadratic inequality with one variable may be in one of the following forms:

$$ax^2 + bx + c < 0 \qquad ax^2 + bx + c > 0$$

$$ax^2 + bx + c \leq 0 \qquad ax^2 + bx + c \geq 0$$

Where  $a$ ,  $b$ , and  $c$  are real numbers and  $a \neq 0$

The solution to a quadratic inequality in one variable is a set of values.

**Example 1:** Solve graphically

a)  $x^2 - 2x - 3 \leq 0$

① graph corresponding quadratic function.

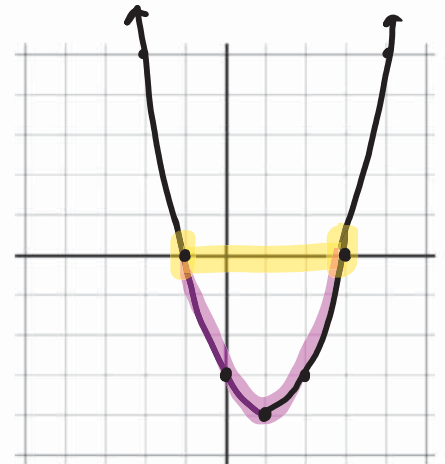
$$y = x^2 - 2x - 3$$

$$y = (x^2 - 2x + \frac{1}{4} - \frac{1}{4}) - 3$$

$$y = (x-1)^2 - 4$$

② Identify the region of the graph that satisfies the inequality.

$x^2 - 2x - 3 \leq 0$  In other words, find the values of  $x$  for which the graph is below the  $x$ -axis.



our graph is below  $x$ -axis between  $x = -1$  and  $x = 3$ .

$$-1 \leq x \leq 3$$

b)  $x^2 - 4x + 3 > 0$

rewrite first

$$x^2 - 4x + 3 > 0$$

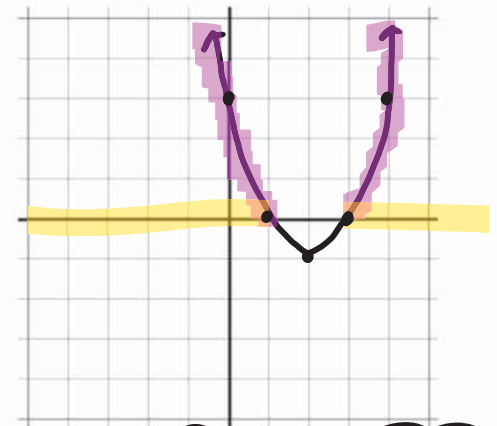
① graph function

$$y = x^2 - 4x + 3$$

$$y = (x^2 - 4x + \frac{4}{4} - \frac{4}{4}) + 3$$

$$y = (x-2)^2 - 1$$

②  $x^2 - 4x + 3 > 0$  what values of  $x$  show the graph is above  $x$ -axis.



$$x < 1 \text{ and } x > 3$$

Example 2: Solve algebraically

a)  $x^2 - 2x - 3 \leq 0$

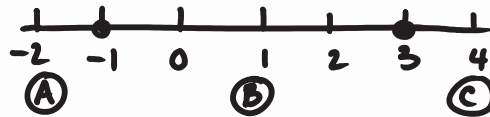
① Find the critical points (critical points are the solutions to the corresponding equation).

$$x^2 - 2x - 3 = 0$$

$$(x-3)(x+1) = 0$$

$$\begin{array}{cc} \downarrow & \downarrow \\ x=3 & x=-1 \end{array}$$

② Set up a number line using the critical points as boundaries.



③ test a point in each boundary (do not use a critical point)

(A) test  $x = -2$

$$\begin{aligned} (-2)^2 - 2(-2) - 3 &\stackrel{?}{\leq} 0 \\ 5 &\stackrel{?}{\leq} 0 \quad \times \end{aligned}$$

(B) test  $x = 0$

$$\begin{aligned} 0^2 - 2(0) - 3 &\stackrel{?}{\leq} 0 \\ -3 &\leq 0 \quad \checkmark \end{aligned}$$

(C) test  $x = 4$

$$\begin{aligned} 4^2 - 2(4) - 3 &\stackrel{?}{\leq} 0 \\ 5 &\stackrel{?}{\leq} 0 \quad \times \end{aligned}$$

$$\boxed{-1 \leq x \leq 3}$$

Final answer

b)  $2x^2 - 12x > -10$   
rewrite

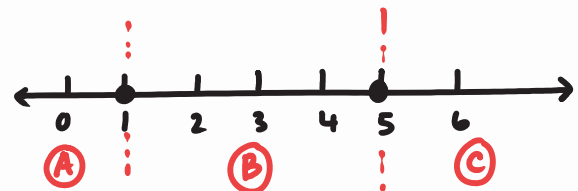
$$2x^2 - 12x + 10 > 0$$

critical points:  $2x^2 - 12x + 10 = 0$

$$2(x^2 - 6x + 5) = 0$$

$$2(x-5)(x-1) = 0$$

$$\begin{array}{cc} \downarrow & \downarrow \\ x=5 & x=1 \end{array}$$



region (A)  $x = 0$

$$\begin{aligned} 2(0)^2 - 12(0) + 10 &\stackrel{?}{>} 0 \\ 10 &\stackrel{?}{>} 0 \quad \checkmark \end{aligned}$$

$$\boxed{x < 1}$$

region (B)  $x = 2$

$$\begin{aligned} 2(2)^2 - 12(2) + 10 &\stackrel{?}{>} 0 \\ 8 - 24 + 10 &\stackrel{?}{>} 0 \\ -6 &\stackrel{?}{>} 0 \quad \times \end{aligned}$$

region (C)  $x = 6$

$$\begin{aligned} 2(6)^2 - 12(6) + 10 &\stackrel{?}{>} 0 \\ 10 &\stackrel{?}{>} 0 \quad \checkmark \end{aligned}$$

$$\boxed{x > 5}$$

Final answer.

Practice: p. 484 # 3a, 4a, 6abc, 7a (Need graph paper)  
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Pre-Calc 11