Assignment 6.3

1. $\sin\theta \cot\theta = \cos\theta$

$\sin\theta \dfrac{\cos\theta}{\sin\theta}$

$\cos\theta \stackrel{\checkmark}{=} \cos\theta$

2. $\cos A \tan A = \sin A$

$\cos A \dfrac{\sin A}{\cos A}$

$\sin A \stackrel{\checkmark}{=} \sin A$

3. $\dfrac{\sin^2\theta + \cos^2\theta}{\cos\theta} = \sec\theta$

$\dfrac{1}{\cos\theta}$

$\sec\theta \stackrel{\checkmark}{=} \sec\theta$

4. $\dfrac{1 + \sin\alpha}{\sin\alpha} = 1 + \csc\alpha$

$\dfrac{1}{\sin\alpha} + \dfrac{\sin\alpha}{\sin\alpha}$

$\csc\alpha + 1 \stackrel{\checkmark}{=} 1 + \csc\alpha$

5. $\dfrac{\sin\alpha - 1}{\cos\alpha} = \tan\alpha - \sec\alpha$

$\dfrac{\sin\alpha}{\cos\alpha} - \dfrac{1}{\cos\alpha}$

LHS $\stackrel{\checkmark}{=} \dfrac{\sin\alpha - 1}{\cos\alpha}$

6. $1 - \sin B \cos B \tan B = \cos^2 B$

$1 - \sin B \cos B \dfrac{\sin B}{\cos B}$

$1 - \sin^2 B$

$\cos^2 B \stackrel{\checkmark}{=} \cos^2 B$

7. $\sin\alpha + \cos\alpha \cot\alpha = \csc\alpha$

$\dfrac{\sin\alpha}{\sin\alpha} \cdot \sin\alpha + \cos\alpha \dfrac{\cos\alpha}{\sin\alpha}$

$\dfrac{\sin^2\alpha}{\sin\alpha} + \dfrac{\cos^2\alpha}{\sin\alpha}$

$\dfrac{\sin^2\alpha + \cos^2 x}{\sin\alpha}$

$\dfrac{1}{\sin\alpha}$

$\csc\alpha \stackrel{\checkmark}{=} \csc\alpha$

8. $1 - 2\sin^2 x = 2\cos^2 x - 1$

$1 - 2(1 - \cos^2 x)$

$1 - 2 + 2\cos^2 x$

$-1 + 2\cos^2 x$

or

$2\cos^2 x - 1 \stackrel{\checkmark}{=}$ RHS

9. $\cos\theta(\csc\theta - \sec\theta) = \cot\theta - 1$

$\cos\theta\csc\theta - \cos\theta\sec\theta$

$\cos\theta \cdot \frac{1}{\sin\theta} - \cancel{\cos\theta} \cdot \frac{1}{\cancel{\cos\theta}}$

$\frac{\cos\theta}{\sin\theta} - 1$

$\cot\theta - 1$ = RHS

10. $\csc\beta(\csc\beta + \cot\beta) = \frac{1}{(1-\cos\beta)} \cdot \frac{(1+\cos\beta)}{(1+\cos\beta)}$

$\frac{1 + \cos\beta}{1 + \cancel{\cos\beta} - \cancel{\cos\beta} - \cos^2\beta}$

$\frac{1 + \cos\beta}{1 - \cos^2\beta}$

$\frac{1 + \cos\beta}{\sin^2\beta}$

$\frac{1}{\sin^2\beta} + \frac{\cos\beta}{\sin^2\beta}$

$\csc^2\beta + \frac{\cos\beta}{\sin\beta} \cdot \frac{1}{\sin\beta}$

$\csc^2\beta + \cot\beta\csc\beta$

LHS $= \csc\beta(\csc\beta + \cot\beta)$

11. $\sin^4 x - \cos^4 x = 2\sin^2 x - 1$

$(\sin^2 x + \cos^2 x)(\sin^2 x - \cos^2 x)$

$(1)(\sin^2 x - \cos^2 x)$

$\sin^2 x - (1 - \sin^2 x)$

$\sin^2 x - 1 + \sin^2 x$

$2\sin^2 x - 1$ = RHS

12. $\tan^4\theta - \sec^4\theta = 1 - 2\sec^2\theta$

$(\tan^2\theta + \sec^2\theta)(\tan^2\theta - \sec^2\theta)$

$(\tan^2\theta + \sec^2\theta)(-1)$

$-(\sec^2\theta - 1) - \sec^2\theta$

$-2\sec^2\theta + 1$ = RHS

or

$1 - 2\sec^2\theta$

13. $\dfrac{\sin\beta+\tan\beta}{1+\cos\beta} = \tan\beta$

$\dfrac{\frac{\cos\beta}{\cos\beta}\cdot\sin\beta + \frac{\sin\beta}{\cos\beta}}{1+\cos\beta}$

$\dfrac{\frac{\cos\beta\sin\beta + \sin\beta}{\cos\beta}}{1+\cos\beta}$

$\dfrac{\frac{\sin\beta(\cos\beta+1)}{\cos\beta}}{1+\cos\beta}$

$\dfrac{\sin\beta(\cos\beta+1)}{\cos\beta}\cdot\dfrac{1}{(1+\cos\beta)}$

$\dfrac{\sin\beta}{\cos\beta}$

$\tan\beta = \text{RHS}$

14. $\sec\theta + \tan\theta = \dfrac{\cos\theta}{(1-\sin\theta)}\cdot\dfrac{(1+\sin\theta)}{(1+\sin\theta)}$

$\dfrac{\cos\theta + \cos\theta\sin\theta}{1+\sin\theta-\sin\theta-\sin^2\theta}$

$\dfrac{\cos\theta + \cos\theta\sin\theta}{1-\sin^2\theta}$

$\dfrac{\cos\theta + \cos\theta\sin\theta}{\cos^2\theta}$

$\dfrac{1 + \sin\theta}{\cos\theta}$

$\dfrac{1}{\cos\theta} + \dfrac{\sin\theta}{\cos\theta}$

$\text{LHS} = \sec\theta + \tan\theta$

15. $(1+\csc x)(1-\sin x) = \cot x\cos x$

$1 - \sin x + \csc x - \csc x\sin x$ | $\dfrac{\cos x}{\sin x}\cdot\cos x$

$1 - \sin x + \csc x - \dfrac{1}{\sin x}\cdot\sin x$ | $\dfrac{\cos^2 x}{\sin x}$

$1 - \sin x + \csc x - 1$

$\dfrac{\sin x}{\sin x}\cdot -\sin x + \dfrac{1}{\sin x}$

$\dfrac{-\sin^2 x + 1}{\sin x}$

$\dfrac{\cos^2 x}{\sin x} = \text{RHS}$

16. $(1+\tan\theta+\sec\theta)^2 = 2(1+\sec\theta)(\tan\theta+\sec\theta)$

LHS RHS

LHS

$(1+\tan\theta+\sec\theta)(1+\tan\theta+\sec\theta)$

$1+\tan\theta+\sec\theta+\tan\theta+\tan^2\theta+\tan\theta\sec\theta+\sec\theta+\sec\theta\tan\theta+\sec^2\theta$

$1+\tan^2\theta=\sec^2\theta$

$1+2\tan\theta+2\sec\theta+2\tan\theta\sec\theta+\tan^2\theta+\sec^2\theta$

$2\tan\theta+2\sec\theta+2\tan\theta\sec\theta+\sec^2\theta+\sec^2\theta$

$2\tan\theta+2\sec\theta+2\tan\theta\sec\theta+2\sec^2\theta$

RHS

$2(1+\sec\theta)(\tan\theta+\sec\theta)$

$2(\tan\theta+\sec\theta+\sec\theta\tan\theta+\sec^2\theta)$

$2\tan\theta+2\sec\theta+2\sec\theta\tan\theta+2\sec^2\theta$

LHS = RHS

17. $(1+\sec\theta)(\sec\theta-1) = \dfrac{\sin\theta\ \sec\theta}{\cos\theta\ \csc\theta}$

$\sec\theta - 1 + \sec^2\theta - \sec\theta$

$\sec^2\theta - 1$

$\tan^2\theta$

$\dfrac{\sin\theta \cdot \frac{1}{\cos\theta}}{\cos \cdot \frac{1}{\sin\theta}}$

$\dfrac{\frac{\sin\theta}{\cos\theta}}{\frac{\cos\theta}{\sin\theta}}$

$\frac{\sin\theta}{\cos\theta} \cdot \frac{\sin\theta}{\cos\theta}$

$\frac{\sin^2\theta}{\cos^2\theta}$

LHS $=$ $\tan^2\theta$

18. $(\csc\theta - 1)(1+\csc\theta) = \dfrac{\csc\theta\cos\theta}{\sec\theta\sin\theta}$

$\csc\theta + \csc^2\theta - 1 - \csc\theta$

$\csc^2\theta - 1$

$\cot^2\theta$

$\dfrac{\frac{1}{\sin\theta} \cdot \cos\theta}{\frac{1}{\cos\theta} \cdot \sin\theta}$

$\dfrac{\frac{\cos\theta}{\sin\theta}}{\frac{\sin\theta}{\cos\theta}}$

$\frac{\cos\theta}{\sin\theta} \cdot \frac{\cos\theta}{\sin\theta}$

$\frac{\cos^2\theta}{\sin^2\theta}$

LHS $=$ $\cot^2\theta$

24 $$\frac{\sec^2\theta(1+\csc\theta)-\tan\theta(\sec\theta+\tan\theta)}{\csc\theta(1+\sin\theta)}-1=0$$

$$\frac{\sec^2\theta+\sec^2\theta\csc\theta-\tan\theta\sec\theta-\tan^2\theta}{\csc\theta(1+\sin\theta)}-1=0$$

$$\frac{\sec^2\theta-\tan^2\theta+\sec^2\theta\csc\theta-\tan\theta\sec\theta}{\csc\theta(1+\sin\theta)}-1=0$$

$$\frac{1+\frac{1}{\cos^2\theta}\cdot\frac{1}{\sin\theta}-\frac{\sin\theta}{\cos\theta}\cdot\frac{1}{\cos\theta}}{\csc\theta(1+\sin\theta)}-1=0$$

$$\frac{1+\frac{1}{\cos^2\theta\sin\theta}-\frac{\sin\theta}{\cos^2\theta}\cdot\frac{\sin\theta}{\sin\theta}}{\csc\theta(1+\sin\theta)}-1=0$$

$$\frac{1+\frac{1-\sin^2\theta}{\cos^2\theta\sin\theta}}{\csc\theta(1+\sin\theta)}-1=0$$

$$\frac{1+\frac{\cos^2\theta}{\cos^2\theta\sin\theta}}{\csc\theta(1+\sin\theta)}-1=0$$

$$\frac{1+\frac{1}{\sin\theta}}{\csc\theta(1+\sin\theta)}-1=0$$

$$\frac{\frac{\sin\theta}{\sin\theta}+\frac{1}{\sin\theta}}{\csc\theta(1+\sin\theta)}-1=0$$

$$\frac{\frac{1}{\sin\theta}(\sin\theta+1)}{\csc\theta(1+\sin\theta)}-1=0$$

$$\frac{\csc\theta}{\csc\theta}-1=0$$

$$1-1=0$$

$$0=0$$

(23) $\dfrac{1+\sin\theta}{\sin\theta-1} + \dfrac{1+\cos\theta}{\cos\theta-1} = 0$

$\dfrac{(\cos\theta-1)}{(\cos\theta-1)} \cdot \dfrac{1+\sin\theta}{\sin\theta-1} + \dfrac{1+\cos\theta}{\cos\theta-1} \cdot \dfrac{(\sin\theta-1)}{(\sin\theta-1)}$

$\dfrac{\cos\theta+\cos\theta\sin\theta-1-\sin\theta}{\cos\theta\sin\theta-\cos\theta-\sin\theta+1} + \dfrac{\sin\theta-1+\cos\theta\sin\theta-\cos\theta}{\cos\theta\sin\theta-\cos\theta-\sin\theta+1}$

$\dfrac{2\cos\theta\sin\theta-2}{\cos\theta\sin\theta-\cos\theta-\sin\theta+1}$

$2\cos\theta \cdot \dfrac{1}{\cos\theta} - 2$

$\dfrac{2-2}{\cos\theta\sin\theta-\cos\theta-\sin\theta+1}$

$\dfrac{0}{\cos\theta\sin\theta-\cos\theta-\sin\theta+1}$

0

0

LS RS

(22) $\dfrac{\tan\theta}{\tan\theta + \sin\theta} = \dfrac{1-\cos\theta}{\sin^2\theta}$

$\dfrac{\frac{\sin\theta}{\cos\theta}}{\frac{\sin\theta}{\cos} + \sin\theta \cdot \frac{\cos\theta}{\cos\theta}}$

$\dfrac{\frac{\sin\theta}{\cos\theta}}{\frac{\sin\theta + \sin\theta\cos\theta}{\cos\theta}}$

$\dfrac{\frac{\sin\theta}{\cos\theta}}{\frac{\sin\theta(1+\cos\theta)}{\cos\theta}}$

$\dfrac{\sin\theta}{\cos\theta} \cdot \dfrac{\cos\theta}{\sin\theta(1+\cos\theta)}$

$\dfrac{1}{1+\cos\theta} \cdot \dfrac{1-\cos\theta}{1-\cos\theta}$

$\dfrac{1-\cos\theta}{1-\cos^2\theta}$

$\dfrac{1-\cos\theta}{\sin^2\theta}$ $\qquad$ $\dfrac{1-\cos\theta}{\sin^2\theta}$

L.S = R.S

(21) $\dfrac{\sec\theta}{1-\cos\theta} = \dfrac{\sec\theta+1}{\sin^2\theta}$

$\dfrac{(1+\cos\theta)}{(1+\cos\theta)}\dfrac{\sec\theta}{1-\cos\theta}$

$\dfrac{\sec\theta + \sec\theta\cos\theta}{1-\cos^2\theta}$

$\dfrac{\sec\theta + \sec\theta\cos\theta}{\sin^2\theta}$

$\dfrac{\sec\theta + \frac{1}{\cos\theta}\cdot\cos\theta}{\sin^2\theta}$

$\dfrac{\sec\theta + 1}{\sin^2\theta}$

L.S

$\dfrac{\sec\theta+1}{\sin^2\theta}$

R.S

(20) $\dfrac{\sin\theta + \cos\theta}{\sec\theta + \tan\theta} + \dfrac{\cos\theta - \sin\theta}{\sec\theta - \tan\theta} = 2 - 2\sin^2\theta \sec\theta$

$\left(\dfrac{\sec\theta - \tan\theta}{\sec\theta - \tan\theta}\right) \cdot \dfrac{\sin\theta + \cos\theta}{\sec\theta + \tan\theta} + \dfrac{\cos\theta - \sin\theta}{\sec\theta - \tan\theta} \cdot \dfrac{\sec\theta + \tan\theta}{\sec\theta + \tan\theta}$

$\dfrac{\sec\theta\sin\theta + \sec\theta\cos\theta - \sin\theta\tan\theta - \cos\theta\tan\theta}{\sec^2\theta - \tan^2\theta} + \dfrac{\cos\theta\sec\theta + \cos\theta\tan\theta - \sin\theta\sec\theta - \sin\theta\tan\theta}{\sec^2\theta - \tan^2\theta}$

$\dfrac{2\sec\theta\cos\theta - 2\sin\theta\tan\theta}{1}$

$2 \cdot \dfrac{1}{\cos\theta} \cdot \cos\theta - 2\sin\theta \cdot \dfrac{\sin\theta}{\cos\theta}$

$2 - 2\sin^2\theta \cdot \dfrac{1}{\cos\theta}$

$2 - 2\sin^2\theta \cdot \sec\theta$

L.S = R.S $\quad 2 - 2\sin^2\theta \sec\theta$

(19) $\dfrac{\sin\theta\cos\theta}{1+\cos\theta} - \dfrac{\sin\theta}{1-\cos\theta} = -(\cot\theta\cos\theta + \csc\theta)$

$$\frac{1-\cos\theta}{1-\cos\theta}\cdot\frac{\sin\theta\cos\theta}{1+\cos\theta} - \frac{\sin\theta}{1-\cos\theta}\cdot\frac{1+\cos\theta}{1+\cos\theta}$$

$$\frac{\sin\theta\cos\theta - \sin\theta\cos^2\theta}{1-\cos^2\theta} - \frac{(\sin\theta + \sin\theta\cos\theta)}{1-\cos^2\theta}$$

$$\frac{-\sin\theta\cos^2\theta - \sin\theta}{1-\cos^2\theta}$$

$$\frac{-\sin\theta(\cos^2\theta + 1)}{1-\cos^2\theta}$$

$$= \frac{-\sin\theta(\cos^2\theta+1)}{\sin^2\theta}$$

$$= \frac{-(\cos^2\theta+1)}{\sin\theta}$$

$-(\cot\theta\cos\theta + \csc\theta$

$-(\frac{\cos\theta}{\sin\theta}\cdot\cos\theta + \frac{1}{\sin$

$$\frac{-(\cos^2\theta+1)}{\sin\theta}$$

$$\frac{-(\cos^2\theta+1)}{\sin\theta}$$

L.S = R.S.