Ch. 3 Review - Solutions

1. a) no ; this is a radical function
   b) yes ; 4th degree polynomial function
   c) yes ; 3rd degree polynomial function
   d) yes ; 1st degree polynomial function

2. a) $h(x) = x^4 - 3x^2 + 5x$
   quartic
   up in Q2 and Q1
   up to 4 x-intercepts
   $h(0) = 0^4 - 3(0)^2 + 5(0)$
   $h(0) = 0$
   y-int is $(0,0)$

   b) $p(x) = -x^3 + 5x^2 - x + 4$
   cubic
   up in Q2 , down in Q4
   up to 3 x-intercepts
   $p(0) = -(0)^3 + 5(0)^2 - 0 + 4$
   $p(0) = 4$
   y-int is $(0,4)$

   c) $y = 3x - 2$
   linear
   down in Q3 , up in Q1
   up to one x-intercept
   $y = 3(0) - 2$
   $y = -2$
   y-int is $(0,-2)$

   d) $y = -2x^2 - 4$
   quadratic
   down in Q3 and Q4
   up to 2 x-intercepts
   $y = -2(0)^2 - 4$
   $y = -4$
   y-int is $(0,-4)$

   e) $t(x) = 2x^5 - 3x^3 + 1$
   quintic
   down in Q3, up in Q1
   up to 5 x-intercepts
   $t(0) = 2(0)^5 - 3(0)^3 + 1$
   $t(0) = 1$
   y-int. is $(0,1)$

3. a) $(x^3 + 9x^2 - 5x + 3) \div (x-2)$
   $P(2) = (2)^3 + 9(2)^2 - 5(2) + 3$
   $P(2) = 8 + 36 - 10 + 3$
   $P(2) = 37$
   Remainder $= R = 37$

$$
\begin{array}{r|l}
 & x^2 + 11x + 17 \\
\hline
x - 2 & x^3 + 9x^2 - 5x + 3 \\
 & -(x^3 - 2x^2) \\
 & \quad 11x^2 - 5x \\
 & \quad -(11x^2 - 22x) \\
 & \qquad 17x + 3 \\
 & \qquad -(17x - 34) \\
 & \qquad\quad 37
\end{array}
$$

Remainder

$$\frac{x^3 + 9x^2 - 5x + 3}{x-2} = x^2 + 11x + 17 + \frac{37}{x-2}$$

   b) $(2x^3 + x^2 - 2x + 1) \div (x+1)$
   $P(-1) = 2(-1)^3 + (-1)^2 - 2(-1) + 1$
   $P(-1) = -2 + 1 + 2 + 1$
   $P(-1) = 2$
   $R = 2$

$$
\begin{array}{r|rrrr}
+1 & 2 & 1 & -2 & 1 \\
 & \downarrow & & & \\
- & & 2 & -1 & -1 \\
\hline
\times & 2 & -1 & -1 & 2
\end{array}
$$

Remainder

$$\frac{2x^3 + x^2 - 2x + 1}{x+1} = 2x^2 - x - 1 + \frac{2}{x+1}$$

   c) $(-8x^4 - 4x + 10x^3 + 15) \div (x+1)$
   $P(-1) = -8(-1)^4 - 4(-1) + 10(-1)^3 + 15$
   $P(-1) = -8 + 4 - 10 + 15$
   $P(-1) = 1$
   $R = 1$

$$
\begin{array}{r|l}
 & -8x^3 + 18x^2 - 18x + 14 \\
\hline
x + 1 & -8x^4 + 10x^3 + 0x^2 - 4x + 15 \\
 & -(-8x^4 - 8x^3) \\
 & \quad 18x^3 + 0x^2 \\
 & \quad -(18x^3 + 18x^2) \\
 & \qquad -18x^2 - 4x \\
 & \qquad -(-18x^2 - 18x)
\end{array}
$$

$$\frac{-8x^4 - 4x + 10x^3 + 15}{x+1} = -8x^3 + 18x^2 - 18x + 14 + \frac{1}{x+1}$$

$$14x + 15$$
$$-(14x + 14)$$
$$1$$

Remainder

4. a) $f(x) = x^4 + Kx^3 - 3x - 5$ $\div$ by $x - 3$ $R = -14$

$(3)^4 + K(3)^3 - 3(3) - 5 = -14$

$81 + 27K - 9 - 5 = -14$

$67 + 27K = -14$

$27K = -81$

$K = -3$

b) $f(x) = x^4 - 3x^3 - 3x - 5$ $\div$ by $x + 3$

$f(-3) = (-3)^4 - 3(-3)^3 - 3(-3) - 5$

$f(-3) = 81 + 81 + 9 - 5$

$f(-3) = 166$

$R = 166$

5. $P(x) = 4x^3 - 3x^2 + bx + 6$ same remainder when divided by $(x-1)$ and $(x+3)$

$P(1) = 4(1)^3 - 3(1)^2 + b(1) + 6$

$P(1) = 4 - 3 + b + 6$

$P(1) = 7 + b$

$P(-3) = 4(-3)^3 - 3(-3)^2 + b(-3) + 6$

$P(-3) = -108 - 27 - 3b + 6$

$P(-3) = -129 - 3b$

$P(1) = P(-3)$

$7 + b = -129 - 3b$

$4b = -136$

$b = -34$

6. a) $x^3 - 4x^2 + x + 6$ factors of 6 $\pm 1, \pm 2, \pm 3, \pm 6$

$x = -1$ works $(-1)^3 - 4(-1)^2 - 1 + 6$

$-1 - 4 - 1 + 6$

0

so $x+1$ is a factor!

1	1	-4	1	6
−	↓	1	-5	6
×	1	-5	6	0

$(x+1)\underbrace{(x^2 - 5x + 6)}_{\text{factor}}$

$x^3 - 4x^2 + x + 6 = (x+1)(x-2)(x-3)$

b) $-4x^3 - 4x^2 + 16x + 16$

factor out -4 first

$-4(x^3 + x^2 - 4x - 4)$ factors of -4 $\{\pm 1, \pm 2, \pm 4\}$

try $x = -1$ $(-1)^3 + (-1)^2 - 4(-1) - 4$

$-1 + 1 + 4 - 4$

$x+1$ is a factor! 0

1	1	1	-4	-4
−	↓	1	0	-4
×	1	0	-4	0

$-4(x+1)\underbrace{(x^2 - 4)}$

$-4(x+1)\underbrace{(x^2-4)}_{\text{factor (diff of squares)}}$

$$-4x^3-4x^2+16x+16 = -4(x+1)(x+2)(x-2)$$

c) $x^4-4x^3-x^2+16x-12$ factors of -12 $\{\pm1, \pm2, \pm3, \pm4, \pm6, \pm12\}$

try $x=1$ $(1)^4-4(1)^3-(1)^2+16(1)-12=0$

so $x-1$ is a factor

-1	1	-4	-1	16	-12
$-$	↓	-1	3	4	-12
$\times$	1	-3	-4	12	0

$(x-1)\underbrace{(x^3-3x^2-4x+12)}_{\text{factor}}$ factors of 12 $\{\pm1, \pm2, \pm3, \pm4, \pm6, \pm12\}$

try $x=2$ $(2)^3-3(2)^2-4(2)+12=0$

so $x-2$ is a factor

-2	1	-3	-4	12
$-$	↓	-2	2	12
$\times$	1	-1	-6	0

$(x-1)(x-2)\underbrace{(x^2-x-6)}_{\text{factor}}$

$$x^4-4x^3-x^2+16x-12 = (x-1)(x-2)(x-3)(x+2)$$

7. $(x^3+4x^2-2kx+3) \div (x+3)$ $R=0$

$(-3)^3+4(-3)^2-2k(-3)+3=0$

$-27+36+6k+3=0$

$12+6k=0$

$k=-2$

8. a) $y=(x+1)(x-2)(x+3)$

x-intercepts

$(-1,0)$

$(2,0)$

$(-3,0)$

degree

3

end behavior

down in Q3

up in Q1

zeros and multiplicity

$x=-1$, $x=2$, $x=-3$ } each have mult. of 1

y-int.

value of y when $x=0$

$y=(0+1)(0-2)(0+3)$

$y=(1)(-2)(3)$

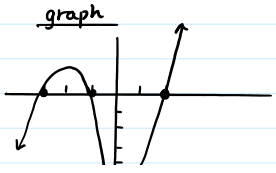
$y=-6$ or $(0,-6)$

intervals

positive : $-3<x<-1$

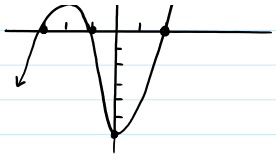
$x>2$

graph



positive : $-3 < x < -1$
$x > 2$

negative : $x < -3$
$-1 < x < 2$



b) $y = (x-3)(x+2)^2$

x - intercepts	degree	end behavior
$(3,0)$	3	down in Q3
$(-2,0)$		up in Q1

zeros and multiplicity
$x = 3$ mult. of 1
$x = -2$ } mult. of 2

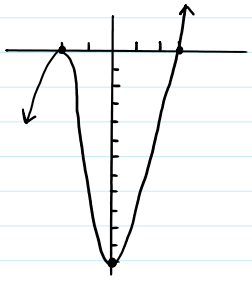
y-int.
$y = (0-3)(0+2)^2$
$y = (-3)(4)$
$y = -12$ or $(0,-12)$

intervals

positive : $x > 3$

negative : $x < -2$
$-2 < x < 3$

graph



c) $g(x) = x^4 - 16x^2$
$g(x) = x^2(x^2 - 16)$
$g(x) = x^2(x-4)(x+4)$

x - intercepts	degree	end behavior
$(0,0)$	4	up in Q2 & Q1
$(4,0)$		
$(-4,0)$		

zeros and multiplicity
$x = 0$ mult. of 2
$x = 4$ } mult. of 1 ea.
$x = -4$

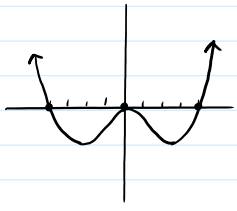
y - int
$y = 0$ or $(0,0)$

intervals

positive : $x < -4$
$x > 4$

negative : $-4 < x < 0$
$0 < x < 4$

graph



9. a) $f(x) = (x+3)^2(x+1)$

b) $f(x) = -(x+1)(x-2)^3$

10. a) $x = -2$
$x = -1$
$x = 3$ mult. of 2

$f(x) = (x+2)(x+1)(x-3)^2$ or $f(x) = -(x+2)(x+1)(x-3)^2$

b) $(2, 24)$

$\uparrow x \quad \uparrow y$

$$
\begin{aligned}
f(x) &= a(x+2)(x+1)(x-3)^2 \\
24 &= a(2+2)(2+1)(2-3)^2 \\
24 &= a(4)(3)(-1)^2 \\
24 &= 12a \\
2 &= a
\end{aligned}
$$

$f(x) = 2(x+2)(x+1)(x-3)^2$