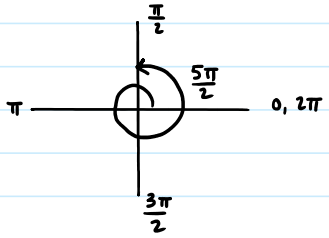
Monday, March 6, 2023 2:34 PM

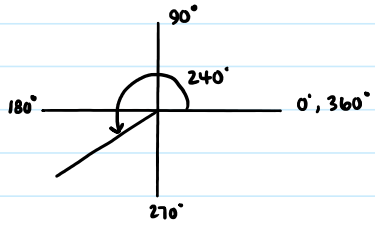
Ch. 4 Review - Solutions

1. a)



$$\frac{5\cancel{\pi}}{\cancel{2}} \times \frac{\overset{90^\circ}{\cancel{180^\circ}}}{\cancel{\pi}} = 5 \cdot 90^\circ = 450^\circ$$

b)



$$\overset{\div 60}{240^\circ} \times \frac{\pi}{\underset{\div 60}{180^\circ}} = \frac{4\pi}{3}$$

2. a) $185^\circ \times \frac{\pi}{180^\circ}$

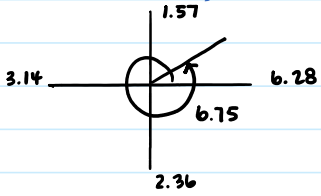
$= 3.23$

b) $\frac{3\cancel{\pi}}{10} \times \frac{180^\circ}{\cancel{\pi}}$

$= \frac{540^\circ}{10}$

$= 54^\circ$

3. a) 6.75 (radians)

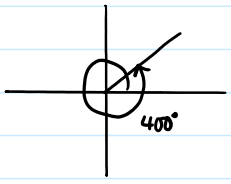


$6.75 + 2\pi = 13.03$

$6.75 - 2\pi(2) = -5.81$

$6.75 \pm 2\pi n, \; n \in N$

b) 400°

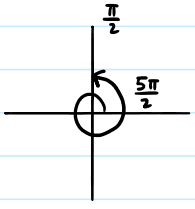


$400 + 360^\circ = 760^\circ$

$400 - 360^\circ(2) = -320^\circ$

$400 \pm 360^\circ n, \; n \in N$

c) $\frac{5\pi}{2}$

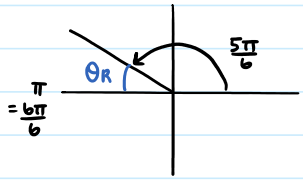


$\frac{5\pi}{2} + 2\pi = \frac{5\pi}{2} + \frac{4\pi}{2} = \frac{9\pi}{2}$

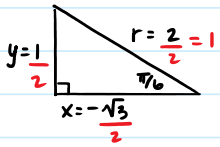
$\frac{5\pi}{2} - 2\pi(2) = \frac{5\pi}{2} - \frac{8\pi}{2} = -\frac{3\pi}{2}$

$\frac{5\pi}{2} \pm 2\pi n, \; n \in N$

4. a) $P\left(\frac{5\pi}{6}\right) = (x, y)$

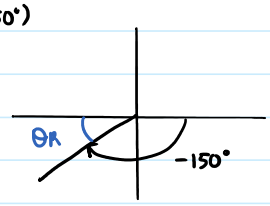


$\theta_R = \frac{\pi}{6}$

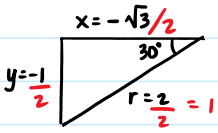


$P\left(\frac{5\pi}{6}\right) = \left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

b) $P(-150°)$

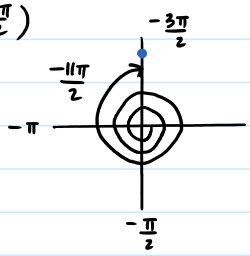


$\theta_R = 30°$



$P(-150°) = \left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

c) $P\left(-\frac{11\pi}{2}\right)$

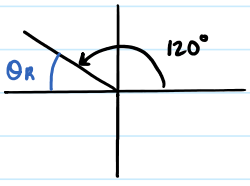


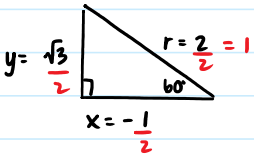
$0, -2\pi$

$-\frac{4\pi}{2}$ $\quad -\frac{8\pi}{2}$

$P\left(-\frac{11\pi}{2}\right) = (0, 1)$

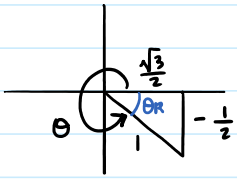
d) $P(120°)$





$P(120°) = \left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

5. a) $P(\theta) = \left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

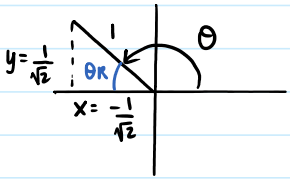


$\theta_R = \frac{\pi}{6}$

$\theta = 2\pi - \frac{\pi}{6}$

$\theta = \frac{11\pi}{6}$

b) $P(\theta) = \left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$



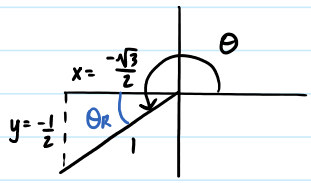
$\theta_R = \frac{\pi}{4}$

$\theta = \pi - \frac{\pi}{4}$

$\theta = \frac{3\pi}{}$

$\theta = \frac{2\pi}{4}$

6. a) $P(\theta) = \left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

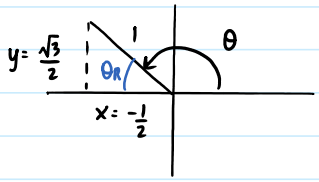


$\theta_R = 30°$

$\theta = 180° + 30°$

$\theta = 210°$

b) $P(\theta) = \left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

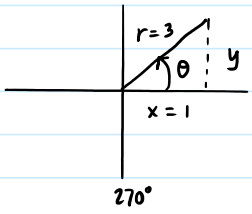


$\theta_R = 60°$

$\theta = 180° - 60°$

$\theta = 120°$

7. $\cos\theta = \frac{1}{3}$ $\quad 0° \le \theta < 270°$



$1^2 + y^2 = 3^2$

$y^2 = 8$

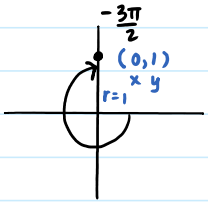
$y = \sqrt{8}$ or $2\sqrt{2}$

$\sin\theta = \frac{\sqrt{8}}{3}$ $\quad \csc\theta = \frac{3}{\sqrt{8}}$

$\cos\theta = \frac{1}{3}$ $\quad \sec\theta = \frac{3}{1}$ or 3

$\tan\theta = \frac{\sqrt{8}}{1}$ or $\sqrt{8}$ $\quad \cot\theta = \frac{1}{\sqrt{8}}$

8. a) $\sin\left(-\frac{3\pi}{2}\right)$

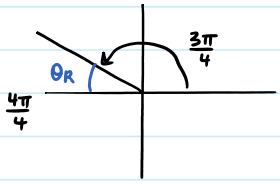


$\frac{y}{r}$

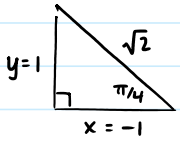
$\sin\left(-\frac{3\pi}{2}\right) = \frac{1}{1}$

$\sin\left(-\frac{3\pi}{2}\right) = 1$

b) $\cos\left(\frac{3\pi}{4}\right)$

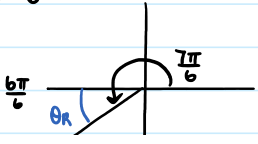


$\theta_R = \frac{\pi}{4}$

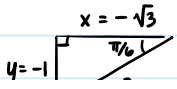


$\cos\left(\frac{3\pi}{4}\right) = \frac{-1}{\sqrt{2}}$

c) $\cot\frac{7\pi}{6}$

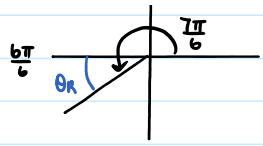


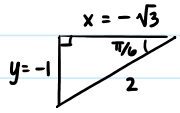
$\theta_R = \frac{\pi}{6}$



$\cot\frac{7\pi}{6} = \frac{-\sqrt{3}}{-1}$

$\cot\frac{7\pi}{6} = \sqrt{3}$

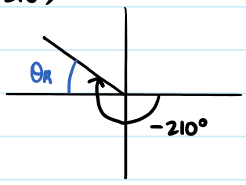




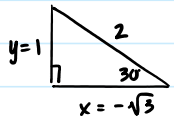
$\cot \frac{6}{\ } = -1$

$\cot \frac{7\pi}{6} = \sqrt{3}$

d) $\sec(-210°)$

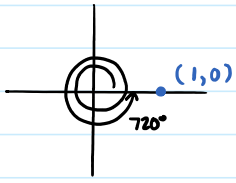


$\theta_R = 30°$



$\sec(-210°) = -\frac{2}{\sqrt{3}}$

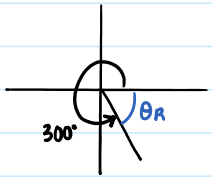
e) $\tan 720°$



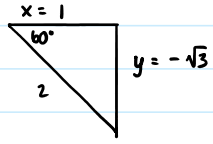
$\tan 720° = \frac{0}{1}$

$\tan 720° = 0$

f) $\csc 300°$

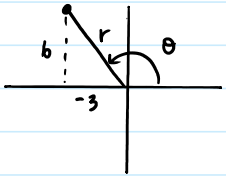


$\theta_R = 60°$



$\csc 300° = -\frac{2}{\sqrt{3}}$

9.



$(-3)^2 + b^2 = r^2$

$9 + 36 = r^2$

$45 = r^2$

$\sqrt{45} = r$

or $3\sqrt{5} = r$

$\sin\theta = \frac{b}{\sqrt{45}}$ or $\frac{2}{\sqrt{5}}$

$\cos\theta = \frac{-3}{\sqrt{45}}$ or $\frac{-1}{\sqrt{5}}$

$\tan\theta = \frac{b}{-3} = -2$

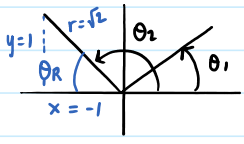
$\csc\theta = \frac{\sqrt{45}}{6}$ or $\frac{\sqrt{5}}{2}$

$\sec\theta = \frac{-\sqrt{45}}{3}$ or $-\sqrt{5}$

$\cot\theta = -\frac{1}{2}$

10. a) $\csc\theta = \sqrt{2}$ $[0°, 360°)$

$\sin\theta = \frac{1}{\sqrt{2}}$



$\theta_R = 45°$

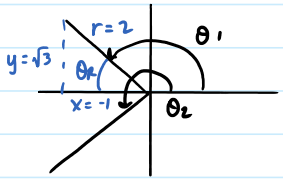
$\theta_1 = 45°$

$\theta_2 = 180° - 45°$

$\theta_2 = 135°$

b) $2\cos\theta + 1 = 0$; $0 \le \theta < 2\pi$

$\cos\theta = -\frac{1}{2}$



$\theta_R = \frac{\pi}{3}$

$\theta_1 = \pi - \frac{\pi}{3}$

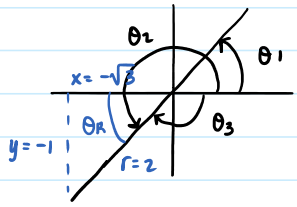
$\theta_1 = \frac{2\pi}{3}$

$\theta_2 = \pi + \frac{\pi}{3}$

$\theta_2 = \frac{4\pi}{3}$

c) $\sqrt{3}\tan\theta - 1 = 0$; $-180^\circ \le \theta < 360^\circ$

$\tan\theta = \frac{1}{\sqrt{3}}$



$\theta_R = 30^\circ$

$\theta_1 = 30^\circ$

$\theta_2 = 180^\circ + 30^\circ$

$\theta_2 = 210^\circ$

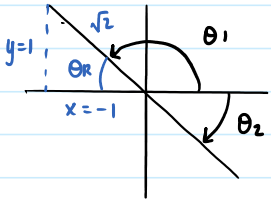
$\theta_3 = -(180^\circ - 30^\circ)$

$\theta_3 = -150^\circ$

d) $\cot\theta + 1 = 0$; $-\pi \le \theta < \pi$

$\cot\theta = -1$

$\tan\theta = -1$



$\theta_R = \frac{\pi}{4}$

$\theta_1 = \pi - \frac{\pi}{4}$

$\theta_1 = \frac{3\pi}{4}$

$\theta_2 = -\theta_R$

$\theta_2 = -\frac{\pi}{4}$

11. a) $\sin^2\theta + \sin\theta - 2 = 0$; $0 \le \theta < 2\pi$

$(\sin\theta + 2)(\sin\theta - 1) = 0$

$\sin\theta = -2$

$\theta = \sin^{-1}(-2)$

no solution

(checked w/ calc.)

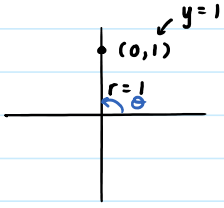
$\sin\theta = 1$

$\theta = \sin^{-1}(1)$

$\theta = 1.571$

or

$= \theta\frac{\pi}{2}$



b) $\tan^2\theta + 3\tan\theta = 0$; $0^\circ \le x < 360^\circ$

$\tan\theta(\tan\theta + 3) = 0$

$\tan\theta = 0$

$\theta = \tan^{-1}(0)$

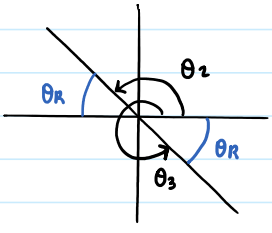
$\theta_1 = 0^\circ$

$\tan\theta = -3$

$\theta = \tan^{-1}(-3)$

$\theta = -71.565^\circ$

$\theta_R = 71.565^\circ$



$\theta_2 = 180^\circ - 71.565^\circ$

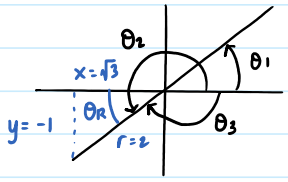
$\theta_3 = 360^\circ - 71.565^\circ$

$\theta_2 = 180° - 71.565°$ $\qquad$ $\theta_3 = 360° - 71.565°$

$\theta_2 = 108.435°$ $\qquad$ $\theta_3 = 288.435°$

c) $\sqrt{3}\tan\theta - 1 = 0$; $-180° \le \theta < 360°$

$\tan\theta = \dfrac{1}{\sqrt{3}}$ $\begin{matrix} y \\ x \end{matrix}$ } special Δ



$\theta_R = 30°$

$\theta_1 = 30°$ $\qquad$ $\theta_2 = 180° + 30°$ $\qquad$ $\theta_3 = -(180° - 30°)$

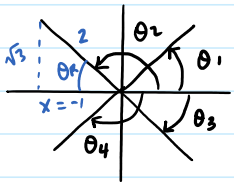
$\theta_2 = 210°$ $\qquad$ $\theta_3 = -150°$

d) $\sec^2\theta - 4 = 0$; $[-\pi, \pi] \rightarrow -\pi \le \theta \le \pi$

$\sqrt{\sec^2\theta} = \sqrt{4}$

$\sec\theta = \pm 2$

$\cos\theta = \pm \dfrac{1}{2}$



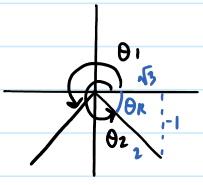
$\theta_R = \dfrac{\pi}{3}$

$\theta_1 = \dfrac{\pi}{3}$ $\qquad$ $\theta_2 = \pi - \dfrac{\pi}{3}$

$\theta_2 = \dfrac{2\pi}{3}$

$\theta_3 = -\dfrac{\pi}{3}$ $\qquad$ $\theta_4 = -\dfrac{2\pi}{3}$

12. a) $\sin\theta = -\dfrac{1}{2}$ (radians)



$\theta_R = \dfrac{\pi}{6}$

$\theta_1 = \pi + \dfrac{\pi}{6}$ $\qquad$ $\theta_2 = 2\pi - \dfrac{\pi}{6}$

$\theta_1 = \dfrac{7\pi}{6}$ $\qquad$ $\theta_2 = \dfrac{11\pi}{6}$

general solution : $\theta = \dfrac{7\pi}{6} \pm 2\pi n, \; n \in N$

and

$\theta = \dfrac{11\pi}{6} \pm 2\pi n, \; n \in N$

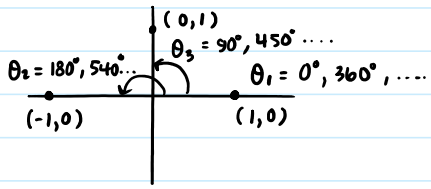
b) $\sin\theta = \sin^2\theta$ (degrees)

$0 = \sin^2\theta - \sin\theta$

$0 = \sin\theta(\sin\theta - 1)$

$\sin\theta = \dfrac{0}{1}$ $\begin{matrix} y \\ r \end{matrix}$ $\qquad$ $\sin\theta = \dfrac{1}{1}$ $\begin{matrix} y \\ r \end{matrix}$

$\sin\theta = \frac{0}{1}\ \frac{y}{r}$ $\qquad$ $\sin\theta = \frac{1}{1}\ \frac{y}{r}$



general solution :

$\theta = 0^\circ \pm 360^\circ n,\ n \in N$

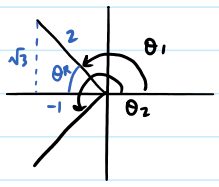
$\theta = 90^\circ \pm 360^\circ n,\ n \in N$

$\theta = 180^\circ \pm 360^\circ n,\ n \in N$

c) $\sec\theta + 2 = 0$ (degrees)

$\sec\theta = -2$

$\cos\theta = -\frac{1}{2}$



$\theta_R = 60^\circ$

$\theta_1 = 180^\circ - 60^\circ$ $\qquad$ $\theta_2 = 180^\circ + 60^\circ$

$\theta_1 = 120^\circ$ $\qquad$ $\theta_2 = 240^\circ$

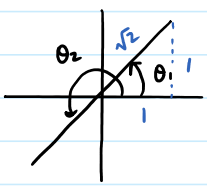
general solution :

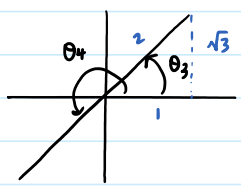
$\theta = 120^\circ \pm 360^\circ n,\ n \in N$

$\theta = 240^\circ \pm 360^\circ n,\ n \in N$

d) $(\tan\theta - 1)(\tan\theta - \sqrt{3}) = 0$ (radians)

$\tan\theta = \frac{1}{1}$ $\qquad$ $\tan\theta = \frac{\sqrt{3}}{1}$





$\theta_R = \frac{\pi}{4}$

$\theta_1 = \frac{\pi}{4}$ $\qquad$ $\theta_2 = \pi + \frac{\pi}{4}$

$\theta_2 = \frac{5\pi}{4}$

$\theta_R = \frac{\pi}{3}$

$\theta_3 = \frac{\pi}{3}$ $\qquad$ $\theta_4 = \pi + \frac{\pi}{3}$

$\theta_4 = \frac{4\pi}{3}$

general solution :

$\theta = \frac{\pi}{4} \pm 2\pi n,\ n \in N$

$\theta = \frac{5\pi}{4} \pm 2\pi n,\ n \in N$

$\theta = \frac{\pi}{3} \pm 2\pi n,\ n \in N$

$\theta = \frac{4\pi}{3} \pm 2\pi n,\ n \in N$