# Chapter 6 Assignment

Name:____________________

Date:____________________

Block:____

1. Simplify the following:

(a) $\dfrac{\csc^2\theta - 2}{\csc^2\theta}$

$= \dfrac{\csc^2\theta}{\csc^2\theta} - \dfrac{2}{\csc^2\theta}$

$= 1 - 2\sin^2\theta$

$= \cos 2\theta$

(b) $\cot^2 x \sin^2 x + \cos^2 x$

$= \dfrac{\cos^2 x \cdot \cancel{\sin^2 x}}{\cancel{\sin^2 x}} + \cos^2 x$

$= 2\cos^2 x$

(c) $\dfrac{\sec\theta - \cos\theta}{\csc\theta - \sin\theta}$

$= \dfrac{\frac{1}{\cos\theta} - \cos\theta \cdot \frac{\cos\theta}{\cos\theta}}{\frac{1}{\sin\theta} - \sin\theta \cdot \frac{\sin\theta}{\sin\theta}}$

$= \dfrac{\frac{1-\cos^2\theta}{\cos\theta}}{\frac{1-\sin^2\theta}{\sin\theta}}$

$= \dfrac{\frac{\sin^2\theta}{\cos\theta}}{\frac{\cos^2\theta}{\sin\theta}}$

$= \dfrac{\sin^2\theta}{\cos\theta} \cdot \dfrac{\sin\theta}{\cos^2\theta}$

$= \dfrac{\sin^3\theta}{\cos^3\theta}$

$= \tan^3\theta$

(d) $\dfrac{2\tan x}{\underbrace{\cos^2 x + \sin^2 x}_{} + \tan^2 x}$

$= \dfrac{2\tan x}{1 + \tan^2 x}$

$= \dfrac{2\tan x}{\sec^2 x}$

$= \dfrac{2\sin x}{\cancel{\cos x}} \cdot \cos^{\cancel{2}} x$

$= 2\sin x\cos x$

$= \sin 2x$

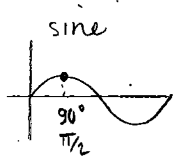
2. Express $2\cos^2 4x - 2\sin^2 4x$ as a single trigonometric function.

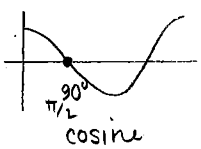
$\cos 2x = \cos^2 x - \sin^2 x$

$= 2\cos 2(4x)$

$= 2\cos 8x$

3. Simplify the following:




(a) $\cos(\alpha + 90°)$

$= \cos\alpha \cancel{\cos 90°} - \sin\alpha \sin 90°$

$= \cos\alpha(0) - \sin\alpha(1)$

$= -\sin\alpha$

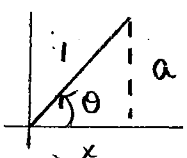
(b) $\sin(\frac{\pi}{2} - \theta)$

$= \sin\frac{\pi}{2}\cos\theta - \cos\frac{\pi}{2}\sin\theta$

$= (1)\cos\theta - (0)\sin\theta$

$= \cos\theta$

4. If $\sin\theta = \frac{a}{1}$ and $\underbrace{0 < \theta < \frac{\pi}{2}}_{\text{quad. I}}$, determine an expression for $\cos(\pi + \theta)$.




$x^2 + a^2 = 1^2$

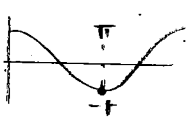
$x^2 = 1 - a^2$

$x = \sqrt{1 - a^2}$

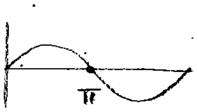
$\cos(\pi + \theta) = \cos\pi\cos\theta - \sin\pi\sin\theta$

$= (-1)\left(\frac{\sqrt{1-a^2}}{1}\right) - (0)(a)$

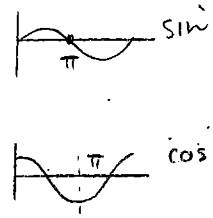
$= -\sqrt{1-a^2}$




cosθ




sinθ



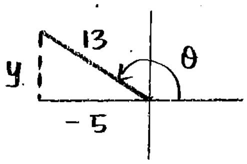

5. If $\sec\theta = \dfrac{-13}{5}$ and $\underbrace{\dfrac{\pi}{2} < \theta < \pi}_{\text{quad 2}}$, determine an expression for $\sin(\theta - \pi)$.

$\cos\theta = -\dfrac{5}{13}$

$\sin(\theta - \pi) = \sin\theta\cos\pi - \cos\theta\sin\pi$

$= \left(\dfrac{12}{13}\right)(-1) - \cancel{\left(-\dfrac{5}{13}\right)(0)}$

$= -\dfrac{12}{13}$




$(-5)^2 + y^2 = (13)^2$

$y = \sqrt{144}$

$y = \pm 12$

6. Solve the following, accurate to 2 decimal places, for $0 \le \theta < 2\pi$

$2\sec^2 x + 5\sec x - 3 = 0$

let $m = \sec x$

$2m^2 + 5m - 3 = 0$

$\underline{6} \times \underline{-1} = -6$

$\underline{6} + \underline{-1} = 5$

$2m^2 + 6m - m - 3 = 0$

$2m(m+3) - 1(m+3) = 0$

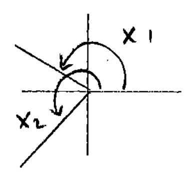
$(m+3)(2m-1) = 0$

$(\sec x + 3)(2\sec x - 1) = 0$

$\sec x = -3$ $\qquad$ $\sec x = \dfrac{1}{2}$

$\cos x = -\dfrac{1}{3}$ $\qquad$ $\underbrace{\cos x = 2}_{\text{no solution}}$

$x = \cos^{-1}\left(-\dfrac{1}{3}\right) = 1.9106$




$x_1 = 1.91$

$x_R = \pi - 1.91 = 1.2310$

$x_2 = \pi + 1.2310$

$x_2 = 4.37$

7. Solve for all possible solutions in radians (general)

$\sin 2x = 2\sin x$

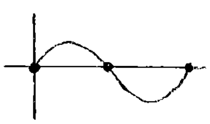
$2\sin x\cos x = 2\sin x$

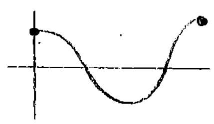
$2\sin x\cos x - 2\sin x = 0$

$2\sin x(\cos x - 1) = 0$

$2\sin x = 0$ $\quad\quad$ $\cos x = 1$

$\sin x = 0$





$x = 0, \pi, 2\pi, \ldots$ $\quad\quad$ $x = 0, 2\pi, \ldots$

$x = 0 + \pi n, \; n \in I$

$x = \pi n, \; n \in I$

8. Prove the following identities.

(a) $\dfrac{\sin 2\theta}{2-2\cos^2\theta} = \cot\theta$

$\dfrac{\cancel{2}\sin\theta\cos\theta}{\cancel{2}(1-\cos^2\theta)}$ $\quad\Big|\quad$ $\dfrac{\cos\theta}{\sin\theta}$

$\dfrac{\cancel{\sin\theta}\cos\theta}{\sin^{\cancel{2}}\theta}$

$\dfrac{\cos\theta}{\sin\theta}$

(b) $\dfrac{\cos\theta}{1-\sin\theta} = \sec\theta + \tan\theta$

$\dfrac{\cos\theta}{(1-\sin\theta)}\cdot\dfrac{1+\sin\theta}{(1+\sin\theta)}$ $\quad\Big|\quad$ $\dfrac{1}{\cos\theta} + \dfrac{\sin\theta}{\cos\theta}$

$\dfrac{\cos\theta(1+\sin\theta)}{1+\sin\theta-\sin\theta-\sin^2\theta}$ $\quad\Big|\quad$ $\dfrac{1+\sin\theta}{\cos\theta}$ $\Big\}$ could keep going to match LHS.

$\dfrac{\cos\theta(1+\sin\theta)}{1-\sin^2\theta}$

$\dfrac{\cancel{\cos\theta}(1+\sin\theta)}{\cos^{\cancel{2}}\theta}$

$\dfrac{1+\sin\theta}{\cos\theta}$

(c) $\tan^2 x - \sin^2 x = \tan^2 x \sin^2 x$

$\frac{\sin^2 x}{\cos^2 x} - \sin^2 x \cdot \frac{\cos^2 x}{\cos^2 x}$

$\frac{\sin^2 x - \sin^2 x \cos^2 x}{\cos^2 x}$

$\frac{\sin^2 x (1 - \cos^2 x)}{\cos^2 x}$

$\tan^2 x \sin^2 x$

(d) $\frac{(\sin\theta + \cos\theta)^2}{\sin 2\theta} = \csc 2\theta + 1$

$\frac{1}{\sin 2\theta} + 1$

$\frac{(\sin\theta + \cos\theta)(\sin\theta + \cos\theta)}{2\sin\theta\cos\theta}$

$\frac{\sin^2\theta + \sin\theta\cos\theta + \sin\theta\cos\theta + \cos^2\theta}{2\sin\theta\cos\theta}$

$\frac{1 + 2\sin\theta\cos\theta}{2\sin\theta\cos\theta}$

$\frac{1}{\sin 2\theta} + 1$

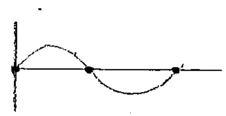
$\csc 2\theta + 1$

9. State all the restrictions for the following:

(a) $\frac{\cot\theta}{1 - \sin\theta}$

numerator: $\cot\theta = \frac{\cos\theta}{\sin\theta}$

$\sin\theta = 0$

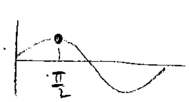


$\theta \neq 0, \pi, 2\pi$, etc ...

$\theta \neq \pi n,\ n \in I$

denom: $1 - \sin\theta = 0$

$1 = \sin\theta$




$\theta \neq \frac{\pi}{2} + 2\pi n,\ n \in I$

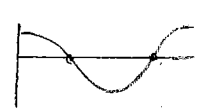
(b) $\frac{\csc\theta}{\cos\theta}$

num: $\csc\theta = \frac{1}{\sin\theta}$

$\sin\theta = 0$

$\theta \neq \pi n,\ n \in I$

denom. $\cos\theta = 0$



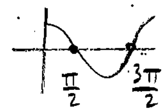
$\theta \neq \frac{\pi}{2} + \pi n,\ n \in I$.

$\theta \neq \frac{\pi}{2} n,\ n \in I$

num: $\sec\theta = \frac{1}{\cos\theta}$

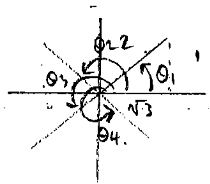
$\cos\theta = 0$

10. State the restrictions for $\dfrac{\sec\theta}{4\sin^2\theta - 1}$ if $0 \le \theta < 2\pi$.




denom: $4\sin^2\theta - 1 \rightarrow (2\sin\theta - 1)(2\sin\theta + 1) = 0$

$\sin\theta = \frac{1}{2}$ $\quad$ $\sin\theta = -\frac{1}{2}$




$\theta_R = \frac{\pi}{6}$

$\theta$

$\theta \neq \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{\pi}{2}, \frac{3\pi}{2}$

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11. Verify $\sin\theta + \cos\theta\cot\theta = \csc\theta$ for $\theta = 30°$.

$\frac{1}{\sin\theta}$

$\sin 30° + \cos 30° \cdot \cot 30° = \csc 30°$

$\frac{1}{2} + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{1}\right) \stackrel{?}{=} \frac{2}{1}$

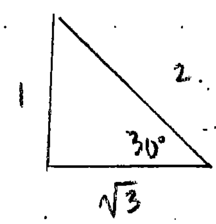
$\frac{1}{2} + \frac{\sqrt{9}}{2}$

$\frac{1}{2} + \frac{3}{2}$

$\frac{4}{2}$

$= 2$

✓




LHS = RHS